TIMING-MISMATCH ANALYSIS IN HIGH-SPEED ANALOG FRONT-END WITH NONUNIFORMLY HOLDING OUTPUT

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ABSTRACT

This paper will present a practical and complete analysis of timing-mismatch effects for high-speed Analog Front-End (AFE) systems with the inherent nonuniformly holding outputs. The analysis reveals first the relationship with traditional impulse-sampled timing-mismatch effects and then its closed-form expressions of the signal-to-noise-ratio (SNR), in terms of the number of channels, signal frequency, and jitter errors. Both timing errors imposed by random clock-jitter and fixed periodic clock-skew will be analyzed. Practical analysis examples for a very high-speed data-converter as well as a AFE filtering will be addressed to illustrate the effectiveness of the derived formula.

1. INTRODUCTION

The rapid evolution of electronic instruments and data communication requires high-speed data conversion channels as well as signal processing units. While the current CMOS process technology limits the achievable speed of the electronics devices, time-interleaved (TI) architectures are one of the most effective ways to boost the maximum speed of the analog interface [1-4], e.g. time-interleaved ADC and DAC for high-speed analog front-end, as shown in Fig.1 [3]. However, due to the nature of TI structures, timing-mismatch of sampling clock phases among different paths can greatly degrade the system performance [5-8], and such jitter noise become much more significant in high-speed systems. This timing-mismatches have generally two different aspects: the first is caused by the random sampling jitter which results in an increased noise floor over all frequencies. The second one is due to the unmatched but fixed periodic timing-skew among different channels, resulting in the image side-bands appearing in frequency locations which are multiple of f_s/M, as shown in Fig. 2, where f_s is the sampling frequency, M is the period of timing-skew, and A_k is the weight of the modulation sidebands.

If the Input signal is sampled by the system with Nonuniform time-interval and the Output signal is played out or represented by its discrete samples at Uniform time instances, it can be classified as **IN-OU** in terms of the I/O sampling process, which is typical in the analog to digital conversion path (timing-mismatch only happens in input signal sampling), e.g. TI ADCs [1] or multirate sampled-data decimators [2]; on the other hand, if the Input signal is sampled by the system with Uniformly spaced time-interval and then the samples are played out at the Output Nonuniformly, then the system can be referred as **IU-ON**, that is the typical case in digital to analog conversion path (timing-mismatch only happens in output signal holding), e.g. TI DACs [3] or multirate sampled-data interpolators [4]. The signal spectra for these two kinds of

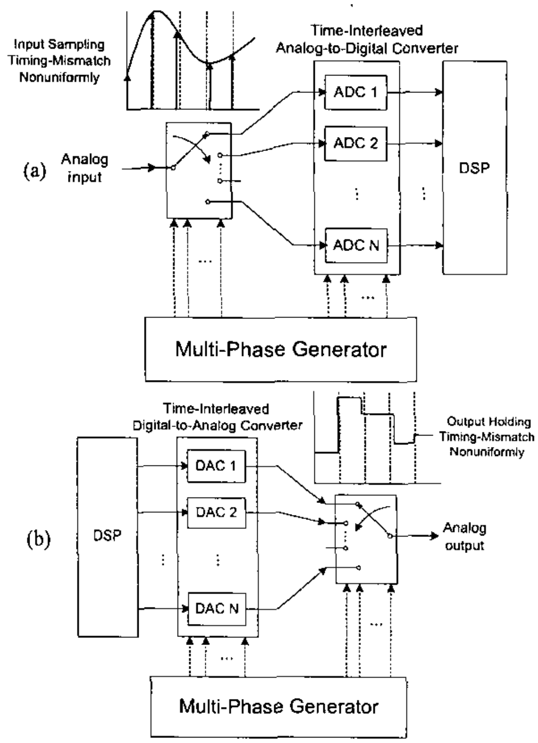

Fig. 1: General time-interleaved data converter structure:
(a) Analog to Digital (b) Digital to Analog Conversion path

processes with Impulse-Sampled (IS) sequence form have been analyzed in [5] and [9], respectively. However in practice, the real output signals are always in Sample-and-Hold (SH) or holding nature in IU-ON process. Especially, such extra holding effect is not simply $\sin(x)/x$ shaped version of the original IS-version spectra due to the effect of output nonuniformly holding waveform as shown in Fig.3. Therefore, the analysis of IU-ON(IS) [9] is not well suited for IU-ON(SH), so this paper will present a complete description of output signal spectrum of IU-ON(SH) process. Besides of this introductory part, a description of the spectra representation for IU-ON(SH) process will be given in part 2, and an interesting spectra correlation between it and IN-OU(IS) will also be described in that section. Second, the closed-form expression of the output signal-to-noise-ratio (SNR) of IU-ON(SH) system will be derived in part 3. In part 4, two practical examples will be illustrated to show the effectiveness of the derived formula, and finally the conclusion is drawn in part 5.

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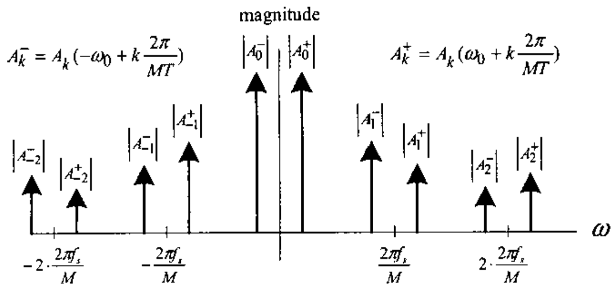


Fig. 2 Typical signal spectrum of sinusoid with timing-skew

2. CORRELATED SPECTRA BETWEEN IN-OU(IS) & IU-ON(SH)

Let us first consider the IN-OU(IS) system, which ideally plays out the nonuniform input samples at uniformly spaced time intervals with impulse-sampled output nature. The output spectra of IN-OU(IS) can be represented as follows [5]:

$$Y(\omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty} A_{1k\_IS}(\omega)\cdot X\left(\omega-k\frac{2\pi}{MT}\right) \tag{1}$$

where

$$A_{1k\_IS}(\omega)=\frac{1}{M}\sum_{m=0}^{M-1}\left(e^{j\omega r_m T}e^{-jkr_m\frac{2\pi}{M}}\right)e^{-jkm\frac{2\pi}{M}} \tag{1a}$$

with $T=1/f_s$ as the nominal sampling period and r_m is a normalized periodic skew timing sequence with period M (M is usually equal to TI path number N), measured in percentage of T.

Also consider the IU-ON(SH) system that ideally plays out the uniform input samples at nonuniformly spaced time intervals with holding nature. The output spectra of IU-ON(SH) can be represented as (1) with different A_k [8]:

$$A_{2k\_SH}(\omega)=\frac{1}{M}\sum_{m=0}^{M-1}H_m(\omega)e^{-j\omega r_m T}e^{-jkm\frac{2\pi}{M}} \tag{2a}$$

$$H_m(\omega)=\frac{2\sin(\omega(1+r_{m+1}-r_m)T/2)}{\omega}e^{-j\omega(1+r_{m+1}-r_m)T/2} \tag{2b}$$

The plots of spectra for IN-OU and IU-ON systems with both IS and SH output are shown in Fig. 4 (with normalized frequency $\alpha=f_o/f_s=0.1$, $M=6$, and standard deviation of r_m equals to 0.16%). From the figure we can see that the IN-OU(SH) is a standard $\sin(x)/x$ shaped version of IN-OU(IS), while the situation in IU-ON system is different (e.g. compare the circled part in figure for two cases) due to the nonuniform holding function as described in (2b).

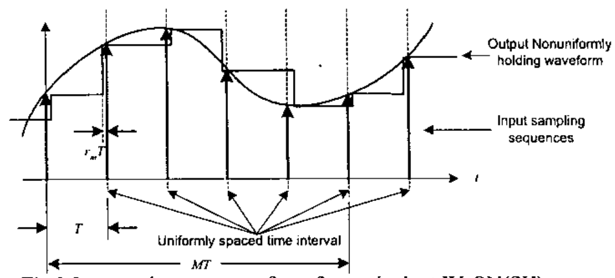


Fig.3 Input and output waveform for equivalent IU-ON(SH) process

Although from (1a), (2a) and (2b) the spectra of IN-OU(IS) and IU-ON(SH) systems are different, with the assumption that $2\pi f_o r_m T<<1$, they will have identical normalized output spectra representation as shown in Fig. 4. The following proves this special correlation.

For a real input sinusoidal signal with frequency $\omega_o=2\pi f_o$, the typical output spectra of both systems can be represented as shown in Fig. 2. In the figure, A_k represents the weighted terms of the modulated sidebands, which are equal to (1a) or (2a), A_k^+ is the terms generated by the positive frequency components of the original sine-wave, while A_k^- is produced by the corresponding negative part. For real signal, $|A_k^+|=|A_{-k}^-|$, so that we can consider only A_k^+ from $k=-\infty$ to $+\infty$, with the sidebands located at the frequency of $\omega=\omega_o+k(2\pi)/(MT)$.

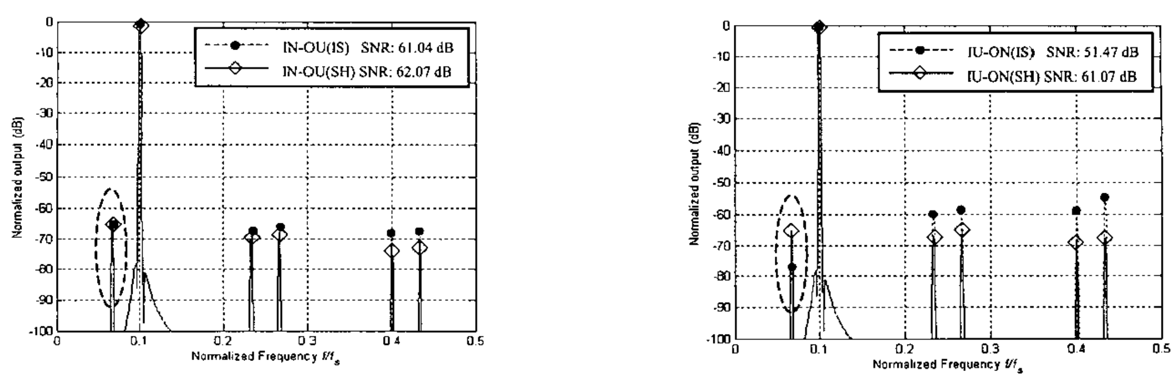
For the IN-OU(IS) system as described by (1a), substituting $\omega=\omega_o+k(2\pi)/(MT)$ into (1a) yields

$$A_{1k\_IS}(\omega_o+k\frac{2\pi}{MT})=\frac{1}{M}\sum_{m=0}^{M-1}\left(e^{j\omega_o r_m T}\right)e^{-jkm\frac{2\pi}{M}} \tag{3}$$

Assuming $\omega_o r_m T=2\pi f_o r_m T<<1$, the magnitude of sidebands components can be expressed as:

$$\left|A_{1k\_IS}(\omega_o+k\frac{2\pi}{MT})\right|\approx\left|\frac{1}{M}\sum_{m=0}^{M-1}(1+j\omega_o r_m T)e^{-jkm\frac{2\pi}{M}}\right|$$

$$=\begin{cases}\dfrac{\omega_o T}{M}\left|\sum_{m=0}^{M-1}r_m e^{-jkm\frac{2\pi}{M}}\right| & \text{for } k\neq 0,\pm M,\ldots \quad (4)\\[2ex] \left|1+j\omega_o T\left(\frac{1}{M}\sum_{m=0}^{M-1}r_m\right)\right|\approx 1 & \text{for } k=0,\pm M,\ldots \quad (5)\end{cases}$$



(a) (b)

Fig. 4 FFT spectra of output sinusoid with normalized frequency of $\alpha=0.1$ for (a) IN-OU and (b) IU-ON processes with both IS and SH output ($M=6$, $\sigma_{rm}=0.16\%$)

For IU-ON(SH) system, the weighted terms of the system are described in (2a) & (2b). Since r_m and $e^{-jkm\frac{2\pi}{M}}$ are periodic with period $m=M$, after simplification of (2a) and (2b) with $\omega=\omega_o+k(2\pi)/(MT)$, (2a) will become

$$A_{2k\_SH}\left(\omega_0+k\frac{2\pi}{MT}\right)=\frac{2\sin(\omega_0 T/2)}{\left(\omega_0+k\frac{2\pi}{MT}\right)M}e^{-j\omega_0 T/2}\sum_{m=0}^{M-1}e^{-j\omega_0 r_m T}e^{-jkr_m\frac{2\pi}{M}}e^{-jkm\frac{2\pi}{M}} \quad (6)$$

By expanding the term $e^{-j\omega_0 r_m T}e^{-jkr_m\frac{2\pi}{M}}$ into Taylor series (r_m <<1) and taking the magnitude, (6) yields

$$\left|A_{2k\_SH}\left(\omega_o+k\frac{2\pi}{MT}\right)\right|=\begin{cases}\frac{2T}{M}\sin(\omega_o T/2)\left|\sum_{m=0}^{M-1}r_m e^{-jkm\frac{2\pi}{M}}\right| & \text{for } k\neq 0,\pm M,\cdots \quad (7)\\ \frac{2}{\omega_o}\sin(\omega_o T/2) & \text{for } k=0,\pm M,\cdots \quad (8)\end{cases}$$

By normalizing the spectra representation (7) with respect to (8), the magnitude of these normalized sideband components of IU-ON(SH) systems is identical to those of IN-OU(IS) systems described by (4), and this completes the prove.

3. THE CLOSED-FORM EXPRESSION FOR SNR

In this section, a closed-form expression for signal-to-noise ratio (SNR) of IU-ON(SH) system is derived. This expression can be used to calculate, for example, the SNR for DAC with nonuniform output sample-and-hold nature. In the following formula derivation, we assume $\omega_o\neq k(2\pi)/(MT)$, which means that the signal (and also the sidebands) is not exactly located at integer multiple of f_s/M.

The SNR can then be found by the following formula [9]:

$$SNR_{2\_SH}=10\log_{10}\left[\frac{\left|A_{20\_SH}(\omega_o)\right|^2}{\sum_{k=1}^{M-1}\left|A_{2k\_SH}\left(\omega_o+k\frac{2\pi}{MT}\right)\right|^2}\right]dB \quad (9)$$

We can determine $\left|A_{20\_SH}(\omega_o)\right|^2$ as follow: i) Assume r_m, $m=0,1,2,\ldots M-1$ *to be* M *independent, identically distributed (i.i.d)* random variables with Gaussian distribution of zero mean and standard deviation of σ_{rm} ($=\sigma_t/T$ where σ_t is the standard deviation of timing jitter with unit of sec), and the characteristic function given by $E[e^{j\omega_0 r_m T}]=e^{-\sigma_{rm}^2\omega_0^2T^2/2}$. ii) Substitute $k=0$ into (6) and by evaluating the expected value of $\left|A_{20\_SH}(\omega_0)\right|^2=A_{20\_SH}(\omega_0)A^*_{20\_SH}(\omega_0)$ it yields:

$$E[\left|A_{20\_SH}(\omega_0)\right|^2]=\frac{4}{\omega_0^2M^2}\sin^2(\omega_0T/2)\sum_{n=0}^{M-1}\sum_{m=0}^{M-1}E\left[e^{-j\omega_0(r_m-r_n)T}\right]$$
$$\approx\frac{4}{\omega_0^2}\sin^2(\omega_0T/2) \quad (10)$$

for small values of r_m such that $2\pi f_o r_m T$<<1. By similar method, we can determine the expected values of the modulated sidebands components $\left|A_{2k\_SH}(\omega_0+k\cdot 2\pi/(MT))\right|^2$ from (6):

$$E\left[\left|A_{2k\_SH}\left(\omega_0+k\cdot\frac{2\pi}{MT}\right)\right|^2\right]\approx\frac{4\sigma_{rm}^2T^2}{M}\sin^2(\omega_0T/2) \quad (11)$$

Substituting (10) & (11) into (9) we will obtain the SNR of an IU-ON(SH) process as

$$SNR_{2\_SH}=20\log_{10}\left(\frac{1}{2\pi a\sigma_{rm}}\right)-10\log_{10}\left(1-\frac{1}{M}\right) \quad (12)$$

Recall that $a=f_o/f_s$ is the normalized frequency of the sinusoid. Actually, this formula is identical to the formula for IN-OU(IS) system [5], which shows that the SNR for these two systems have the same expressions for small jitter errors, thus proving again the special correlation between IN-OU(IS) and IU-ON(SH).

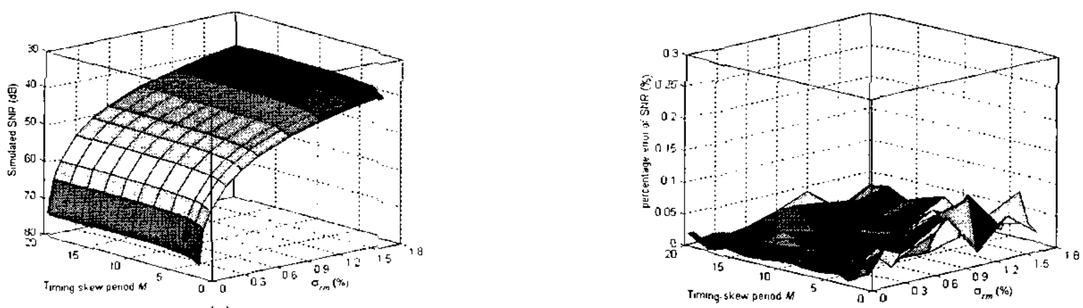
Fig. 5 shows the simulation results using MATLAB (a=0.1) to illustrate the accuracy of the derived formula (12) compared with simulated SNR of IU-ON(SH) system. As shown in Fig. 5(b), the error between output SNR and that predicted by formula (12) is well below 0.1% for reasonable range of SNR, showing that (12) can well estimate the SNR of IU-ON(SH) systems.

4. PRACTICAL APPLICATIONS

We present here two practical examples to illustrate the theories described in the above sections in both random timing jitter and periodic timing-mismatch aspects:

<u>1. Random jitter noise results in increasing noise floor over all frequencies</u>: The random timing jitter is equivalent to a time-skew sequence with period $M\to\infty$. A jittered signal spectrum of M=100 shown in Fig. 6 presents that the modulated sidebands converge to an increased noise floor as M becomes large. The calculated SNR for σ_{rm}=0.1% and 1% with a=0.1 are 64.04 dB and 44.04 dB, respectively, comparable with the simulated 63.85 dB and 44.01dB. Moreover, the SNR increases by 20 dB per decade with respect to the σ_{rm}, thus showing the consistence between the proposed theoretical prediction and practical results.

Consider an 8-bit 8Gsamples/s DAC at the transmission end of a serial-link transceiver described in [3]. To achieve 8-bit accuracy, the SNR subjected to the noise power caused by the



(a) (b)

Fig. 5 (a) Simulated SNR & (b) percentage error between the simulated and calculated SNR of IU-ON(SH) system vs. timing-skew period M and the standard deviation r_m by 10^5 times Monte Carlo Simulations(normalized signal frequency a=0.1)

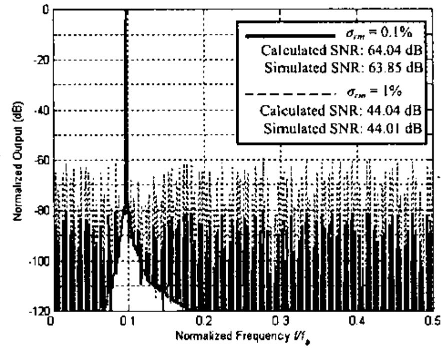


Fig. 6 FFT spectrum of output sinusoid for IU-ON(SH) processes (α=0.1 M=100)

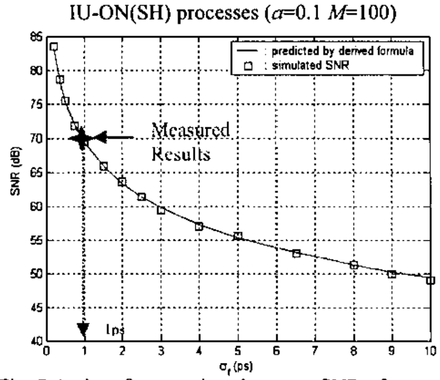


Fig. 7 A plot of comparison between SNR of example 2 obtained by MATLAB simulation and formula calculation (with α=58/320=0.18, M=8)

random jitter generated by the on-chip Phase Lock-Loop must be much higher than 50 dB. Thus, at the Nyquist rate of 4 GHz (α=0.5) the standard deviation σ_{rm} must be smaller than 0.1%, which is equivalent to a time jitter of 0.125 ps calculated using (12). This requirement is critical to be achieved in current CMOS technology, where the measured results show only σ_t of 2.5 ps can be obtained [3].

2. *Periodic timing skew results in modulated sidebands appearing in frequency locations of f_s/M*: This periodic timing skew will appear from unmatched but fixed propagation delay among the TI phase generation paths. Besides, supply variation caused by dI/dt noise can destroy the matching of the rising edges of the TI phases, thus producing periodic timing skew. Consider an example of a very high-frequency (output sampling rate at 320MHz) SC multirate bandpass interpolating filter from [4]. The periodic timing-skew mismatch (M=8) among TI clocks is a dominating factor for the special on-chip multirate phase generator driven by an external master clock. Due to the holding nature at lower input sampling rate of the analog interpolation, the timing-mismatch errors at the input sampling stage are negligible, thus being equivalent to an IU-ON(SH) system. From the derived formula, the standard deviation of the output jitter must be well controlled under the stringent requirement of σ_t<5ps (σ_{rm}<5ps·320MHz=0.16%) to realize SNR>60dB. Simulation results demonstrate that, the worst possible timing-skew can be as large as 100ps which will completely destroy the system performance without special design and care in the implementation of multirate clock generator [8].

Fig. 7 shows the SNR from FFT simulated and calculated by derived formula vs. σ_t with signal frequency of 58MHz and M=8. This plot shows that the simulated result tracks very well the theoretical SNR curve from (12). The ratio between the measured signal and the timing-jitter noise is around 70dB [8], corresponding to a fixed periodic timing-skew σ_t of roughly 1ps, which is well below the requirement of 5 ps. This proves the effectiveness of the various used design and layout techniques [8], and also shows the need of prediction of the allowed timing jitter errors during the design.

5. CONCLUSION

A complete analysis of timing-mismatch effects for high-speed Analog Front-End systems with practical nonuniformly holding output (IU-ON(SH)) is presented in this paper. Due to the fact that the sample-and-hold function is nonuniform in the time-domain, the output spectrum is not simply a shaped version (by $\sin(x)/x$) of that spectra with impulse-sampled output. It has been proved that the spectra of IU-ON(SH) is interestingly similar to that of IN-OU(IS) when timing-jitter is sufficiently small ($2\pi f_o r_m T$<<1). The closed-form expression of the SNR for IU-ON(SH) systems has also been derived in this paper, and two practical examples are followed to illustrate the effectiveness of the derived formula. MATLAB simulation shows that the prediction accuracy of the SNR formula is within a range of 0.1%.

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