# Analysis and Modeling of a Gain-Boosted N-Path Switched-Capacitor Bandpass Filter

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Abstract—It has been studied that, an N-path switched-capacitor (SC) branch driven by an N-phase non-overlapped local oscillator (LO), is equivalent to a tunable parallel-RLC tank suitable for radio-frequency (RF) filtering. This paper proposes a gain-boosted N-path SC bandpass filter (GB-BPF) with a number of sought features. It is based on a transconductance amplifier  $(\mathbf{G_m})$  with an N-path SC branch as its feedback network, offering: 1) double RF filtering at the input and output of the  $G_{\rm m}$  in one step; 2) customized passband gain and bandwidth with input-impedance match; and 3) reduced physical capacitance thanks to the loop gain offered by G<sub>m</sub>. All have been examined using a RLC model of the SC branch before applying the linear periodically time-variant (LPTV) analysis to derive the R, L, and C expressions and analytically study the harmonic selectivity, harmonic folding, and noise. The latter reveals that: 1) the noise due to the switches is notched at the output, allowing smaller switches to save the LO power and 2) the noises due to the source resistance and  $G_{\mathbf{m}}$  are narrowband at the output, reducing the folded noise during harmonic mixing. To study the influence of circuit non-idealities, an intuitive equivalent circuit model is also proposed and verified. The design example is a four-path 0.5-2-GHz GB-BPF simulated with the 65-nm CMOS. It exhibits >11 dB gain, <2.3 dB NF, and +21-dBm out-of-band IIP3 at 150-MHz offset, while consuming just 7 mW of power.

Index Terms—Bandpass filter, bandwidth, CMOS, feedback network, gain boosted, harmonic mixing, input-impedance match, linear periodically time-variant (LPTV), local oscillator (LO), N-path, passive mixer, receiver, radio-frequency (RF) filtering, switched-capacitor (SC), switches, transconductance amplifier.

#### I. INTRODUCTION

HE demand of highly integrated multiband transceivers has driven the development of blocker-tolerant software-defined radios that can avoid the cost (and loss) of the baluns and SAW filters [1]–[3]. The passive-mixer-first receivers [1], [2] achieve a high out-of-band (OB) linearity (IIP3 = +25 dBm) by eliminating the forefront low-noise amplifier (LNA). However, in the absence of radio-frequency (RF) gain, a considerable amount of power is entailed for the local oscillator (LO) to drive up the mixers that must be essentially large (i.e., small on-resistance,  $R_{\rm sw}$ ) for an affordable noise figure (NF < 5 dB). The noise-cancelling receiver in [3] breaks such a NF-linearity

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tradeoff, by noise-cancelling the main path via a high-gain auxiliary path, resulting in better NF (1.9 dB). However, due to the wideband nature of all RF nodes, the passive mixers of the auxiliary path should still be large enough for a small  $R_{\rm sw}$  (10  $\Omega$ ) such that the linearity is upheld (IIP3 = +13.5 dBm). Indeed, it would be more effective to perform filtering at the antenna port.

An N-path switched-capacitor (SC) branch applied at the antenna port [4], [5] corresponds to direct filtering that enhances OB linearity, although the sharpness and ultimate rejection are limited by the capacitor size and non-zero  $R_{\rm sw}$  that are tight tradeoffs with the area and LO power, respectively. Repeatedly adopting such filters at different RF nodes can raise the filtering order, but at the expense of power and area [5], [6].

Active-feedback frequency translation loop [7] is another technique to enhance the area efficiency (0.06 mm²), narrowing RF bandwidth via signal cancellation, instead of increasing any RC time-constant. Still, the add-on circuitry (amplifiers and mixers) penalizes the power (62 mW) and NF (>7 dB). In [8], at the expense of more LO power and noise, the output voltages can be extracted from the capacitors via another set of switches, avoiding the effects of  $R_{\rm sw}$  on the ultimate rejection, but the problem of area remains unsolved.

Recently, an ultra-low-power multiband ZigBee receiver [9] was demonstrated, which features a novel gain-boosted N-path passive mixer to optimize the NF and OB linearity with power. The underlying principle is generalized here for the first time, leading to a *gain-boosted N-path SC bandpass filter (GB-BPF)* with a number of attractive features: 1) tunability of center frequency, passband gain and bandwidth without affecting the input-impedance matching; 2) lower LO power as the pitfall of big  $R_{\rm sw}$  can be leveraged by other design freedoms; and 3) much smaller capacitors for a given bandwidth thanks to the gain-boosting effects.

This paper is organized as follows: Section II introduces the proposed GB-BPF and describes its features via an ideal RLC model first. Linear periodically time-variant (LPTV) analysis is then followed to derive and examine the models of those R, L and C. The analysis of harmonic selectivity, harmonic folding and noise are detailed in Section III, where an equivalent circuit model for studying the influence of non-idealities is included. In Section IV, a simulation design example is given. Finally, the conclusions are drawn in Section V.

#### II. GB-BPF USING AN IDEAL RLC MODEL

The proposed GB-BPF is depicted in Fig. 1(a). It features a transconductance amplifier  $(\mathrm{G_m})$  in the forward path, and an N-path SC branch driven by an N-phase non-overlapped LO in the feedback path. When one of the switches is ON, an in-phase RF voltage  $V_{\mathrm{RF}}$  will appear on the top plate of capacitor  $\mathrm{C_i},$  and induces an amplified anti-phase voltage into its bottom plate.

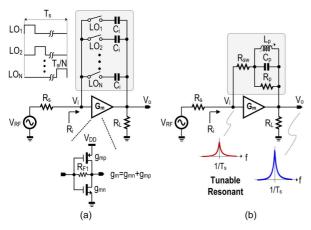


Fig. 1. (a) Proposed gain-boosted N-path SC bandpass filter (GB-BPF) and (b) its equivalent RLC circuit with the LC resonant tunable by the LO.  $R_{\rm sw}$  is the mixer switch's on-resistance.

When the switch is OFF, the amplified version of  $V_{\rm RF}$  will be stored in  $C_i$ . There are three observations: 1) similar to the well-known capacitor-multiplying technique (i.e., Miller effect) in amplifiers, the effective capacitance of  $C_i$  at the input node  $V_i$  will be boosted by the loop gain created by  $G_{\rm m}$ , while it is still  $C_i$  at the output node  $V_o$ . This feature, to be described later, reduces the required  $C_i$  when comparing it with the traditional passive N-path filter. 2) For the in-band signal, the voltages sampled at all  $C_i$  are in-phase summed at  $V_i$  and  $V_o$  after a complete LO switching period  $(T_s)$ , while the OB blockers are cancelled to each other, resulting in double filtering at two RF nodes in one step. 3) As the switches are located in the feedback path, their effects to the OB rejection should be reduced when comparing it with the passive N-path filter.

For simplicity,  $G_m$  is assumed as an inverter amplifier with an effective transconductance of  $g_m$ . It is self-biased by the resistor  $R_{F1}$  and has a finite output resistance explicitly modeled as  $R_L$ . The parasitic effects will be discussed in Section III-C. With both passband gain and resistive input impedance, the GB-BPF can be directly connected to the antenna port for matching with the source impedance  $R_S$ . Around the switching frequency  $(\omega_s)$ , the N-path SC branch is modeled as an  $R_p-L_p-C_p$  parallel network [10] in series with  $R_{sw}$ , where  $L_p$  is a function of  $\omega_s$  and will resonate with  $C_p$  at  $\omega_s$  [Fig. 1(b)]. The expressions of  $R_p$ ,  $L_p$  and  $C_p$  will be derived in Section II-C. Here, the filtering behavior and -3-dB bandwidth at  $V_i$  and  $V_o$  will be analyzed.

#### A. RF Filtering at $V_i$ and $V_o$

With  $V_{RF}$  centered at frequency  $f_{RF}=f_s=\omega_s/2\pi,\,L_P$  and  $C_p$  are resonated out, yielding an input resistance  $R_i|_{@fs}$  that can be sized to match  $R_S$  for the in-band signal

$$R_{\rm i}|_{@f_{\rm s}} = \frac{(R_{\rm p} + R_{\rm sw})//R_{\rm F1} + R_{\rm L}}{1 + g_{\rm m}R_{\rm L}} = R_{\rm S}.$$
 (1)

For the OB blockers located at  $f_{\rm RF}=f_{\rm s}\pm\Delta f_{\rm s},$  either  $L_{\rm p}$  or  $C_{\rm p}$  will become a short circuit when  $\Delta f_{\rm s}$  is large enough

$$\begin{split} R_{\rm i}|_{@f_{\rm s}\pm\Delta f_{\rm s}} &= \frac{(R_{\rm sw}//R_{\rm F1}) + R_{\rm L}}{1 + g_{\rm m}R_{\rm L}} \\ &\approx \frac{R_{\rm sw} + R_{\rm L}}{1 + g_{\rm m}R_{\rm L}} \approx \frac{R_{\rm sw}}{g_{\rm m}R_{\rm L}} + \frac{1}{g_{\rm m}} \end{split} \tag{2}$$

where  $R_{\rm F1}\gg R_{\rm sw}$  and  $g_{\rm m}R_{\rm L}\gg 1$  are applied and reasonable to simplify (2). To achieve stronger rejection of OB blockers at

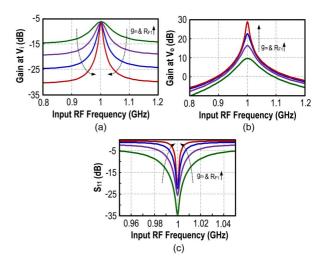


Fig. 2. Simulated (a) gain at  $V_{\rm i}$ , (b) gain at  $V_{\rm o}$  and (c)  $S_{11}$ , showing how  $g_{\rm m}$  and  $R_{\rm F1}$  tune the in-band gain and bandwidth while keeping the in-band  $S_{11}$  well below -20 dB.

 $V_i$ , a small  $R_i|_{@f_s\pm\Delta f_s}$  is expected. Unlike the traditional passive N-path filter where the OB rejection is limited by  $R_{sw}$  [10], [15], this work can leverage it with three degrees of freedom:  $g_m$ ,  $R_L$  and  $R_{sw}$ . As a GB-BPF at the forefront of a receiver, a large  $g_m$  is important to lower the NF of itself and its subsequent circuits. As an example, with  $g_m=100~mS$ , the product of  $g_m R_L$  can reach 8 V/V with  $R_L=80~\Omega$ . Thus, if  $R_{sw}=20~\Omega$  is assumed, we obtain  $R_i|_{@f_s\pm\Delta f_s}\approx12.5~\Omega$ , which is only 62.5% of  $R_{sw}$ . If  $g_m$  is doubled (i.e., more power) while maintaining the same  $g_m R_L$ ,  $R_i|_{@f_s\pm\Delta f_s}$  is reduced to 7.5  $\Omega$ . Another way to trade the OB rejection with power is to adopt a multistage amplifier as  $G_m$ , which can potentially decouple the limited  $g_m R_L$ -product of a single-stage amplifier in nanoscale CMOS.

OB filtering not only happens at  $V_{\rm i}$ , but also  $V_{\rm o}$ . Hence, with one set of switches, double filtering is achieved in this work, leading to higher power and area efficiency than the traditional cascade design (i.e., two SC branches separately applied for  $V_{\rm i}$  and  $V_{\rm o}$ ) as described in [5]. Likewise, the gain at  $V_{\rm o}$  at the resonance can be found as

$$A_{\rm vo}|_{@f_{\rm s}} = \frac{V_{\rm o}}{V_{\rm RF}} = \frac{R_{\rm L}(1 - g_{\rm m}R_{\rm T})}{2R_{\rm S}(1 + g_{\rm m}R_{\rm L})} \approx \frac{R_{\rm L}(1 - g_{\rm m}R_{\rm T})}{2R_{\rm S}g_{\rm m}R_{\rm L}}$$
 (3)

where  $R_T = R_{F1}//(R_p + R_{sw})$  and  $g_m R_L \gg 1$  are applied In terms of stability, (3) should be negative or zero, i.e.,  $g_m R_T \leq 1$ . Similarly, the gain at  $V_o$  at  $f_s \pm \Delta f_s$  is derived when  $L_p$  or  $C_p$  is considered as a short circuit

$$\frac{V_{o}}{V_{RF}}\bigg|_{@f_{s} \pm \Delta f_{s}} = \frac{1 - g_{m}R_{sw}}{1 + g_{m}R_{S} + \frac{R_{S}}{R_{L}} + \frac{R_{sw}}{R_{L}}}.$$
 (4)

Interestingly, if  $g_{\rm m}R_{\rm sw}=1$ , the OB filtering is infinite. This is possible because the feedback network is frequency selective, implying that the in-band signal and OB blockers can see different feedback factors. This fact differentiates this circuit from the traditional resistive-feedback wideband LNAs such as [11] that cannot help to reject the OB blockers.

To exemplify, the circuit of Fig. 1(a) is simulated for N=4, using PSS and PAC analyses in SpectreRF. The parameters are:  $R_{\rm sw}=20~\Omega,~R_{\rm L}=80~\Omega,~R_{\rm S}=50~\Omega,~C_{\rm i}=5~pF$  and  $f_{\rm s}=1~GHz.$  As expected, higher selectivity at  $V_{\rm i}$  [Fig. 2(a)] and  $V_{\rm o}$  [Fig. 2(b)] can be observed when  $g_{\rm m}$  (100 to 800 mS) and  $R_{\rm F1}$  (500 to 8  $k\Omega$ ) are concurrently raised, while preserving

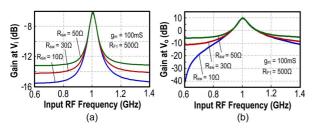


Fig. 3.Simulated (a) gain at  $V_i$ , (b) gain at  $V_o$  under  $R_{\rm sw}=10,30$  and  $50\,\Omega$ .

the in-band  $S_{11}<-20$  dB. [Fig. 2(c)]. Alternatively, when  $R_{\rm sw}$  goes up from 10 to 50  $\Omega,$  with other parameters unchanged, it can be observed that the influence of  $R_{\rm sw}$  to the OB rejection is relaxed at both  $V_{\rm i}$  [Fig. 3(a)] and  $V_{\rm o}$  [Fig. 3(b)], being well-consistent with (2) and (4). When  $R_{\rm sw}=10~\Omega,$  a much stronger OB rejection is due to  $g_{\rm m}R_{\rm sw}$  in (4).

#### B. -3-dB Bandwidth at $V_i$ and $V_o$

At frequency  $f_{RF}=f_s$ , we can write  $V_i/V_{RF}|_{@f_s}=1/2$  when  $R_i=R_s$ . The -3-dB bandwidth is calculated by considering that the  $L_pC_p$  tank only helps shifting the centre frequency of the circuit from DC to  $f_s$ , keeping the same bandwidth as it is without  $L_p$ . If  $R_{sw}$  is neglected and the Miller approximation is applied, the -3-dB passband bandwidth  $(2\Delta f_{i3}\ dB)$  at  $V_i$  can be derived

$$2\Delta f_{i3 \text{ dB}} = \frac{1}{\pi R_e C_i}; \quad C_i \approx (1 + A_{vi})C_p$$
 (5)

where

$$A_{vi} = \frac{V_o}{V_i} = \frac{R_L(1 - g_m R_T)}{R_S(1 + g_m R_T)}$$

Obviously,  $\mathrm{C_p}$  is boosted by a gain factor  $\mathrm{A_{vi}}$ , which should be 15 to 20 dB in practice. Thus, a large  $\mathrm{A_{vi}}$  can be used to improve the area efficiency, consistent with the desire of higher selectivity OB filtering, as shown in Fig. 2(a) and (b). Passive N-path filters [10] do not exhibit this advantageous property and the derived  $\mathrm{C_p}$  is also different. In Section III-D, an intuitive eqivalent circuit model of Fig. 1(a) will be given for a more complete comparison with the traditional architecture.

At  $V_{\rm o},$  the -3-dB passband bandwidth  $(2\Delta f_{\rm o3~dB})$  can be derived next, assuming  $R_{\rm sw}=0$  for simplicity. The gain from  $V_{\rm RF}$  to  $V_{\rm o}$  at frequency  $f_{\rm s}-\Delta f_{\rm o3~dB}$  is given by

$$A_{vo}|_{@f_s - \Delta f_o 3} dB = \frac{V_o}{V_{RF}} = \frac{R_L(1 - g_m Z_T)}{2R_S(1 + g_m R_L)}$$
 (6)

where

$$Z_{\rm T} = jL_{\rm eff}//R_{\rm F1}//R_{\rm p} \text{ and } L_{\rm eff} \approx \frac{\omega_{\rm s} - \Delta\omega_{\rm o3} \text{ dB}}{2\frac{\Delta\omega_{\rm o3} \text{ dB}}{\omega}}L_{\rm p}.$$
 (7)

From the definition of -3-dB passband bandwidth

$$\frac{\left|A_{\text{vo}|_{@f_s}}\right|}{\left|A_{\text{vo}|_{@f_s} - \Delta f_o 3} \text{ dB}\right|} = \frac{|1 - g_m R_{\text{FP}}|}{|1 - g_m Z_T|} = \sqrt{2}$$
 (8)

where  $A_{\rm vo|_{\mathfrak{G}_{f_s}}}$  is the voltage gain at the resonant frequency, while  $R_{\rm FP}=R_{\rm F1}//R_{\rm P}$ . Substituting (6), (7) into (8), (9) is obtained after simplification

$$L_{\text{eff}} = \frac{\sqrt{g_{\text{m}}^2 R_{\text{FP}}^2 - 2g_{\text{m}} R_{\text{FP}} - 1} \times R_{\text{FP}}}{g_{\text{m}} R_{\text{FP}} - 1} \approx R_{\text{FP}}.$$
 (9)

Substituting (9) into (7),  $\Delta\omega_{o3}$  dB becomes

$$\Delta\omega_{\rm o3~dB} = \frac{\omega_{\rm s}^2}{2\frac{\rm L_{\rm eff}}{\rm L_p} + \omega_{\rm s}} \approx \frac{\omega_{\rm s}^2}{2\frac{\rm L_{\rm eff}}{\rm L_p}} = \frac{1}{2\rm L_{\rm eff}C_p} = \frac{1}{2\rm R_{\rm FP}C_p}. \tag{10}$$

Finally,  $2\Delta f_{o3} dB$  at  $V_o$  can be approximated as

$$2\Delta f_{o3 \text{ dB}}|_{@V_o} \approx \frac{1}{\pi R_{FP}C_p}$$

C. Derivation of the  $R_p - L_p - C_p$  Model Using the LPTV Analysis

The GB-BPF [Fig. 1(a)] can be classified as a LPTV system. This section derives the  $R_{\rm p}-L_{\rm p}-C_{\rm p}$  model of the gain-boosted N-path SC branch. Similar to [12], [13], the voltage on the SC branch is defined as

$$V_{Ci}(j\omega) = \sum_{n=-\infty}^{\infty} H_{n,RF}(j\omega) V_{RF}(j(\omega - n\omega_s)).$$
 (11)

Here n indicates a harmonic number of  $f_s$ , and  $H_{n,RF}(j\omega)$  is the nth harmonic transfer function associated with the frequency  $nf_s.$  With  $V_{ci}(j\omega)$ , the voltages at  $V_i(j\omega)$  and  $V_o(j\omega)$  can be related to the input RF signal  $V_{RF}(j\omega)$ 

$$V_{i}(j\omega) = \underbrace{V_{RF}(j\omega)\frac{1}{\gamma}\left(\beta\frac{R_{L}}{R_{S}} + H_{0,RF}(j\omega)\right)}_{V_{i,de}} + \underbrace{\frac{1}{\gamma}\sum_{n=-\infty,n\neq0}^{\infty}H_{n,RF}(j\omega)V_{RF}(j(\omega - n\omega_{s}))}_{V_{c}}$$
(12)

and

$$V_{o}(j\omega) = \underbrace{\frac{R_{F1}R_{L}\left(1 - g_{m}R_{sw} + \frac{R_{sw}}{R_{F1}}\right)}{R_{F1}R_{SW} + (R_{F1} + R_{sw})(R_{s} + g_{m}R_{L}R_{s} + R_{L})}}_{V_{o,de}} \times \underbrace{\left[V_{RF}(j\omega) - \frac{H_{0,RF}(j\omega)V_{RF}(j\omega)(1 + g_{m}R_{s})}{\left(1 - g_{m}R_{sw} + \frac{R_{sw}}{R_{F1}}\right)}\right]}_{V_{o,de}} - \underbrace{\frac{R_{F1}R_{L}(1 + g_{m}R_{s})}{R_{F1}R_{SW} + (R_{F1} + R_{sw})(R_{s} + g_{m}R_{L}R_{s} + R_{L})}}_{V_{o,un}} \times \underbrace{\sum_{n = -\infty, n \neq 0}^{\infty} H_{n,RF}(j\omega)V_{RF}\left(j(\omega - n\omega_{s})\right).}_{(13)}$$

where

$$\begin{split} \alpha &= 1 - g_m R_{sw} + \frac{R_{sw}}{R_{F1}} \\ \beta &= 1 + \frac{R_{sw}}{R_L} + \frac{R_{sw}}{R_{F1}} \\ \gamma &= \alpha + \beta \left( \frac{R_L}{R_S} + g_m R_L \right). \end{split}$$

Equations (12) and (13) can be divided into two parts: 1) the desired frequency selectivity (i.e.,  $V_{i,de}$  and  $V_{o,de}$ ) that provides

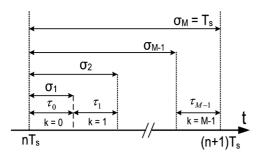


Fig. 4. Time intervals for the state-space analysis.

filtering without frequency translation at the desired input frequency, and 2) the undesired harmonic folding components that might fall in the desired band (i.e.,  $V_{i,un}$  and  $V_{o,un}$ ).

To find  $H_{n,RF}(j\omega)$ , a state-space analysis is conducted. The timing diagram for the analysis is shown in Fig. 4. The timing interval  $nT_s < t < nT_s + T_s$  is divided into M portions (M is the number of the states) and each portion, identified by k, can be represented as  $nT_s + \sigma_k < t < nT_s + \sigma_{k+1}, k = 0, \ldots, M-1$  and  $\sigma_0 = 0$ . During each interval there is no change in the state of the switches, and the network can be considered as a LTI system. During the k interval, linear analysis applied to Fig. 1(a) reveals that the switch on interval k has the following state-space description:

$$\begin{cases}
\frac{C_{i}dv_{C_{i}}(t)}{dt} + \frac{v_{i}(t) - v_{o}(t)}{R_{F_{1}}} = \frac{v_{o}(t)}{R_{L}} + g_{m}v_{i}(t) \\
\frac{v_{RF}(t) - v_{i}(t)}{R_{S}} = \frac{v_{o}(t)}{R_{L}} + g_{m}v_{i}(t) \\
v_{i}(t) = v_{C_{i}}(t) + v_{o}(t) + R_{sw}\frac{C_{i}dv_{C_{i}}(t)}{dt}.
\end{cases} (14)$$

From (14), we obtain

$$\frac{\mathrm{d}v_{\mathrm{Ci}}(t)}{\mathrm{d}t} = \frac{v_{\mathrm{RF}}(t)}{\mathrm{C_i}\mathrm{R_1}} - \frac{v_{\mathrm{Ci}}(t)}{\mathrm{C_i}\mathrm{R_2}}$$
(15)

where

$$\begin{split} R_1 &= \frac{1 + \frac{R_{\rm sw}}{R_{\rm F1}} + \frac{R_{\rm sw} + R_{\rm S}}{R_{\rm L}} + \frac{R_{\rm sw}R_{\rm S}}{R_{\rm F1}R_{\rm L}} + g_{\rm m}R_{\rm S} + \frac{g_{\rm m}R_{\rm sw}R_{\rm S}}{R_{\rm F1}}}{\frac{1}{R_{\rm L}} + g_{\rm m}} \\ R_2 &= \frac{1 + \frac{R_{\rm sw}}{R_{\rm F1}} + \frac{R_{\rm sw} + R_{\rm S}}{R_{\rm L}} + \frac{R_{\rm sw}R_{\rm S}}{R_{\rm F1}R_{\rm L}} + g_{\rm m}R_{\rm S} + \frac{g_{\rm m}R_{\rm sw}R_{\rm S}}{R_{\rm F1}}}{\frac{1}{R_{\rm F1}} + \frac{1}{R_{\rm L}} + \frac{R_{\rm S}}{R_{\rm F1}R_{\rm L}} + \frac{g_{\rm m}R_{\rm S}}{R_{\rm F1}}}. \end{split}$$

By applying the state-space analysis for the circuit in Fig. 1(a), the harmonic transfer function can be derived as

$$H_{n,RF}(j\omega) = \sum_{m=0}^{N-1} e^{-jn\omega_s \sigma_m} H_{n,m}(j\omega)$$

$$H_{n,m}(j\omega) = \frac{\omega_{rc,B}}{\omega_{rc,A} + j\omega} \times \frac{1 - e^{-jn\omega_s \tau_m}}{j2\pi n} + \frac{1 - e^{j(\omega - n\omega_s)(T_S - \tau_m) - jn\omega_s \tau_m}}{\omega_{rc,A} + j\omega} G(j\omega) f_s \quad (16)$$

where

$$G(j\omega) = \frac{e^{j(\omega - n\omega_s)\tau_m} - e^{-\omega_{rc,A}\tau_m}}{e^{j2\pi(\omega - n\omega_s)/\omega_s} - e^{-\omega_{rc,A}\tau_m}} \times \frac{1}{\frac{\omega_{rc,A}}{\omega_{rc,B}} + \frac{j(\omega - n\omega_s)}{\omega_{rc,B}}}$$

and where  $\omega_{\rm rc,A}=1/R_2C_i$  and  $\omega_{\rm rc,B}=1/R_1C_i$ . The above  $H_{\rm n,RF}(j\omega)$  is undefined for n=0, and, for this value, (16) will be defined by the limit when n tends to zero, implying that

$$H_{0,RF}(j\omega) = \frac{\omega_{rc,B}}{\omega_{rc,A} + j\omega} + \frac{1 - e^{j\omega(T_S - \tau_m)}}{\omega_{rc,A} + j\omega} G(j\omega) f_s N \quad (17)$$

where

$$G(j\omega) = \frac{e^{j\omega\tau_{m}} - e^{\omega_{rc,A}\tau_{m}}}{e^{\frac{j2\pi\omega}{\omega_{s}}} - e^{-\omega_{rc,A}\tau_{m}}} \times \frac{1}{\frac{somega_{rc,A}}{\omega_{rc,B}} + \frac{j\omega}{\omega_{rc,B}}}.$$

To find  $R_p$ ,  $H_{0,RF}(j\omega)$  is calculated at  $\omega=nf_s$  with  $\omega_s\gg\omega_{rc,A}$ ,  $\omega_{rc,B}$ , yielding

$$H_{0,RF}(jn\omega_s) = \frac{2N(1-\cos 2\pi nD)}{4D(n\pi)^2} \times \frac{\omega_{rc,B}}{\omega_{rc,A}}$$
(18)

where D=1/N is the duty cycle of the LO. Furthermore, (18) is similar to (15) in [10], except for the added term  $\omega_{\rm rc,B}/\omega_{\rm rc,A}$ . If n=1, N=4 and D=0.25, for a 25%-duty-cycle fourpath LO, (18) becomes

$$H_{0,RF}(j\omega_s) = \frac{8}{\pi^2} \times \frac{R_2}{R_1}.$$
 (19)

Assuming that  $L_p$  is resonant with  $C_p$  at  $\omega_s$ , it implies

$$\begin{cases} \frac{V_{i}-H_{0,RF}(j\omega_{s})V_{RF}-V_{o}}{R_{sw}} = \frac{H_{0,RF}(j\omega_{s})V_{RF}}{R_{p}} \\ \frac{V_{i}-H_{0,RF}(j\omega_{s})V_{RF}-V_{o}}{R_{sw}} + \frac{V_{i}-V_{o}}{R_{F1}} = g_{m}V_{i} + \frac{V_{o}}{R_{L}} \\ \frac{V_{RF}-V_{i}}{R_{s}} = g_{m}V_{i} + \frac{V_{o}}{R_{L}}. \end{cases}$$
(20)

Solving (20), it leads to the desired  $R_P$ 

$$R_{p} = \frac{\eta H_{0,RF} R_{sw}}{\left(\frac{R_{L}R_{FL}}{R_{s}} + \frac{H_{0,RF}}{R_{sw}}\right) \left(1 + \frac{R_{L}}{R_{s}} + g_{m}R_{L}\right) - \left(H_{0,RF} + \frac{R_{L}}{R_{s}}\right) \eta}$$

where

$$\begin{split} R_{\rm FL} &= \frac{1}{R_{\rm L}} + \frac{1}{R_{\rm F1}} + \frac{1}{R_{\rm sw}} \\ \eta &= \frac{1}{R_{\rm sw}} + \frac{1}{R_{\rm F1}} - g_{\rm m} + \frac{R_{\rm L}R_{\rm FL}}{R_{\rm s}} + g_{\rm m}R_{\rm L}R_{\rm FL}. \end{split}$$

Finally, placing the pole around  $\omega_{\rm s}$  in (17), with a value equal to the poles of the transfer function from  $V_{\rm RF}$  to  $V_{\rm Cp}$  of Fig. 1(b), it will lead to the expressions of  $C_{\rm p}$  and  $L_{\rm p}$ 

$$C_{p} = \frac{\gamma_{1} + R_{p}}{2D\omega_{rc,A}\gamma_{1}R_{p}}$$
(21)

$$L_{p} = \frac{\gamma_{1}R_{p}}{D\omega_{rc,A}(\gamma_{1} + R_{p}) - \left(D^{2}\omega_{rc,A}^{2} - \omega_{s}^{2}\right)\gamma_{1}R_{p}C_{p}}$$
(22)

where

$$\begin{split} \alpha_1 &= \frac{1}{R_{sw}} + \frac{1}{R_{F1}} - g_m, \; \gamma_1 = -\frac{\alpha_1 \beta_1 R_{sw}^2}{\beta_1 - 1 - \alpha_1 \beta_1 R_{sw}}, \\ \beta_1 &= \frac{\frac{1}{R_L} + \frac{1}{R_{F1}} + \frac{1}{R_{sw}} + \frac{\alpha_1 R_s}{R_L (1 + g_m R_s)}}{\frac{1}{R_L} + g_m}. \end{split}$$

From (21) and (22),  $C_p$  is irrelevant to the LO frequency  $\omega_s$ , while  $L_p$  is tunable with  $\omega_s$ . Moreover, the term  $D\omega_{rc,A}(\gamma_1+R_P)-(D^2\omega_{rc,A}^2-\omega_s^2)\gamma_1R_pC_p$  in the denominator of (22) renders that the  $L_p//C_p$  resonant frequency shifts slightly away from the center frequency  $\omega_s$ . For  $\omega_s\gg\omega_{rc,A},L_p\approx R_p/\omega_s^2C_p$  is obtained and will resonate out with  $C_p$  at  $\omega_s$ . Then, the frequency responses can be plotted using the derived expressions, and compared with the simulated curves of Fig. 5(a) and (b); showing a good fitting around  $\omega_s$ , and confirming the previous analysis. The small discrepancy arises from the approximation that  $L_p$  will resonate out with  $C_p$  at  $\omega_s$  when deriving  $R_p$  in (20). This effect is smaller at  $V_i$  than at  $V_o$ , due to the gain of the GB-BPF.

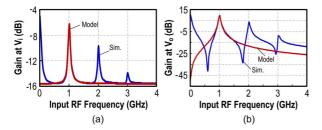


Fig. 5. Comparison between the simulation and the analytic derived model using (21) and (22). (a) Gain at  $V_i$  and (b) gain at  $V_o$ . The parameters are  $R_{\rm sw}=10~\Omega,\,R_{\rm L}=80~\Omega,\,R_{\rm S}=50~\Omega,\,C_i=5~\text{pF},\,g_{\rm m}=100~\text{mS},\,R_{\rm F1}=500~\Omega,\,f_{\rm s}=1~\text{GHz},$  and N=4.

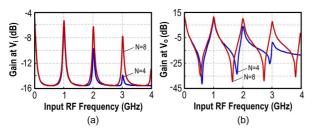


Fig. 6. Simulated responses under N=4 and N=8. (a) Gain at  $V_i$  and (b) gain at  $V_o$ . The responses are consistent with (17) (not plotted).

## III. HARMONIC SELECTIVITY, HARMONIC FOLDING, AND NOISE

#### A. Harmonic Selectivity and Harmonic Folding

Using the harmonic selectivity function  $H_{0,RF}(j\omega)$  from (18), the relative harmonic selectivity is calculated by combining (13) and (18) for  $V_i$  and  $V_o$ . For example, when N=4

$$\frac{V_0(\omega_s)}{V_0(n\omega_s)} = \frac{1 - \frac{8}{\pi^2} \times \frac{R_2}{R_1} \times Constant}{1 - \frac{8}{(n\pi)^2} \times \frac{R_2}{R_1} \times Constant} \approx n^2$$

which matches with the four-path passive mixer [10]. Likewise, using (12) and (18), the harmonic selectivity at  $V_i$  is derived as

$$\frac{V_i(\omega_s)}{V_i(n\omega_s)} \approx \frac{R_L + \frac{8}{\pi^2} \times R_{F1}}{R_L + \frac{8}{(n\pi)^2} \times R_{F1}} < n^2.$$

Obviously, the harmonic selectivity at  $V_i$  is smaller than that at  $V_o$  with the design parameters used here.

The above analysis has ignored the even-order harmonic selectivity which should be considered in single-ended designs. The harmonic selectivity for N=4 and N=8 with a fixed total value of capacitance and  $g_m R_{\rm sw}=1$  are shown in Fig. 6(a) and 6(b), respectively. For N=4,  $V_{\rm o}(3\omega_{\rm s})/V_{\rm o}(\omega_{\rm s})=18.67$  dB and  $V_{\rm i}(3\omega_{\rm s})/V_{\rm i}(\omega_{\rm s})=7.6$  dB, close to the above analysis. Moreover, the relative harmonic selectivity can be decreased by raising N. Furthermore, as derived in (4),  $g_{\rm in}R_{\rm sw}=1$  results in a stronger OB attenuation at far out frequencies that are irrelevant to N. Finally, the bandwidth at  $V_{\rm i}$  and  $V_{\rm o}$  can be kept constant if the total amount of capacitors is fixed under different N. This will be quite explicit when the equivalent circuit will be presented later in Section III-C.

For N=4, the simulated harmonic folding at  $V_i$  and  $V_o$  are shown in Fig. 7(a) and (b), respectively, which obey well (12), (13), and (16) (not plotted). Similar to the N-path passive mixers, the input frequencies around  $k(N\pm1)f_s$  will be folded onto the desired frequency around  $f_s$ . The strongest folding term is from  $3f_s$  when k=1, and will become smaller

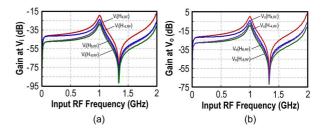


Fig. 7. Simulated harmonic folding effects under N=4. (a) Gain at  $V_i$  and (b) gain at  $V_{\circ}$ . The responses are consistent with (16) (not plotted).

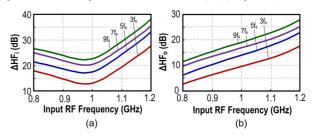


Fig. 8. Simulated harmonic folding gain (normalized) under N=4 (a) at  $V_{\rm i}$  and (b)  $V_{\rm o}.$ 

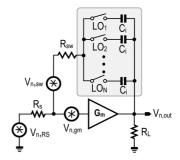


Fig. 9. Equivalent noise model of the GB-BPF.

if k (integer number) is increased. The relative harmonic folding  $\Delta HF_i = 20 \log[V_{i,\mathrm{de}}(j\omega)] - 20 \log[V_{i,\mathrm{un}}(j\omega)]$  and  $\Delta HF_o = 20 \log[V_{o,\mathrm{de}}(j\omega)] - 20 \log[V_{o,\mathrm{un}}(j\omega)]$  are plotted in Fig. 8(a) and (b), respectively. The relative harmonic folding is smaller at  $V_i$  than at  $V_o$ , which is preferable because harmonic folding at  $V_i$  cannot be filtered.

#### B. Noise

The output noises under consideration are the thermal noises from  $R_{\rm s},\,R_{\rm sw}$  and  $G_{\rm m}.$  Since the power spectral density (PSD) of these noise sources are wideband, harmonic folding noise should be considered. The model to derive those noise transfer functions is shown in Fig. 9.

To calculate the noise from  $R_{\rm s}$  to  $V_{\rm o}$  (13) needs to be revised in order to obtain

$$\begin{split} & \frac{\overline{V_{n,out,RS}^{2}}}{= \underbrace{\left| \frac{R_{F1}R_{L} \left( 1 - g_{m}R_{sw} + \frac{R_{sw}}{R_{F1}} \right)}{R_{F1}R_{SW} + (R_{F1} + R_{sw})(R_{s} + g_{m}R_{L}R_{s} + R_{L})} \right|^{2}}_{Part\ A}} \\ & \times \left| V_{n,RS}(j\omega) \right|^{2} \times \left| 1 - \frac{H_{0,RF}(j\omega)(1 + g_{m}R_{s})}{\left( 1 - g_{m}R_{sw} + \frac{R_{sw}}{R_{F1}} \right)} \right|^{2}} \end{split}$$

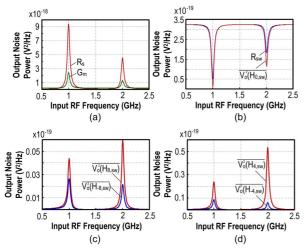


Fig. 10. Simulated output noise power at  $V_o$  due to (a)  $R_{\rm S}$  and  $G_{\rm m}$  and (b)  $R_{\rm sw}$ . The results are consistent with (23), (25), and (27) (not plotted). The output noise power  $\overline{V_o^2(H_0(j\omega))}$  with notch shape of  $R_{\rm sw}$  is plotted in (b) using (25) Part A. The harmonic folding parts  $\overline{V_o^2(H_{\pm 4}(j\omega))}$  and  $\overline{V_o^2(H_{\pm 8}(j\omega))}$  using (25) Part B are plotted in (c) and (d). The parameters are  $R_{\rm sw}=30~\Omega, R_{\rm L}=80~\Omega, R_{\rm S}=50~\Omega, C_{\rm i}=5~\rm pF, g_{\rm m}=100~mS, R_{\rm F1}=500~\Omega, f_{\rm s}=1~\rm GHz, N=4, \overline{V_{\rm n,sw}^2}=4kTR_{\rm sw}=4.968\times10^{-19}~\rm (V^2/Hz), \overline{V_{\rm n,Rs}^2}=4kTR_{\rm s}=8.28\times10^{-19}~\rm (V^2/Hz), and \overline{V_{\rm n,gm}^2}=4kT/g_{\rm m}=1.656\times10^{-19}~\rm (V^2/Hz).$ 

$$+\underbrace{\left|\frac{R_{F1}R_{L}(1+g_{m}R_{s})}{R_{F1}R_{SW}+(R_{F1}+R_{sw})(R_{s}+g_{m}R_{L}R_{s}+R_{L})}\right|^{2}}_{Part B}$$

$$\times\underbrace{\sum_{n=-\infty,n\neq0}^{\infty}\left|H_{n,RF}(j\omega)V_{n,RS}\left(j(\omega-n\omega_{s})\right)\right|^{2}}_{Part B}.$$
(23)

In (23), Part A is the output noise PSD due to  $R_{\rm s}$  without frequency translation, while Part B is due to harmonic folding. Similarly, linear analysis of  $v_{\rm n,sw}(t)$  results in the state-space description

$$\frac{\mathrm{d}v_{\mathrm{Ci}}(t)}{\mathrm{d}t} = \frac{v_{\mathrm{n,sw}}(t)}{\mathrm{C_i}\mathrm{R_1}} - \frac{v_{\mathrm{Ci}}(t)}{\mathrm{C_i}\mathrm{R_2}}$$
(24)

where

$$\begin{split} R_1 &= \frac{-(1 + \alpha_2 R_{sw})}{\alpha_2} \\ R_2 &= - \, R_1 \\ \alpha_2 &= \frac{\left(\frac{1}{R_{F1}} + \frac{1}{R_S} + \frac{R_L}{R_{F1} R_S} + \frac{g_m R_L}{R_{F1}}\right)}{\left(1 + g_m R_L + \frac{R_L}{R_S}\right)} \end{split}$$

with a minus sign in  $R_1.$  Combining (24) with (16) and (17), the output noise PSD transfer function of  $R_{\rm sw}$  from  $V_{\rm n,sw}$  to  $V_{\rm ci}$  [i.e.,  $H_{\rm 0,sw}(j\omega)$ ] and its harmonic folding [i.e.,  $H_{\rm n,sw}(j\omega)$ ] can be derived, leading to the final output noise of PSD to  $V_o$  expressed as

$$\frac{V_{n,\text{out,sw}}^{2}}{V_{n,\text{out,sw}}^{2}} = \frac{\left|V_{n,\text{sw}}(j\omega)\right|^{2} \left|\left(1 + H_{0,\text{sw}}\right)\right|^{2}}{\left|\left(-\frac{R_{S}}{\gamma_{2}R_{L}} - 1 - \frac{R_{\text{sw}}}{\gamma_{2}R_{L}} - \frac{R_{\text{sw}}}{R_{F1}} - \frac{R_{\text{sw}}R_{S}}{\gamma_{2}R_{L}R_{F1}}\right)\right|^{2}}}{P_{\text{art }}A} + \underbrace{\sum_{n=-\infty,n\neq 0}^{\infty} \left|\frac{H_{n,\text{sw}}(j\omega)V_{n,\text{sw}}(j\omega - jn\omega_{s})}{-\frac{R_{S}}{\gamma_{2}R_{L}} - 1 - \frac{R_{\text{sw}}}{\gamma_{2}R_{L}} - \frac{R_{\text{sw}}}{R_{F1}} - \frac{R_{\text{sw}}R_{S}}{\gamma_{2}R_{L}R_{F1}}}\right|^{2}}_{P_{\text{out}}} \right|^{2}}_{P_{\text{out}}}$$
(25)

where

$$\gamma_2 = 1 + g_m R_s$$
.

In (25), Part A is the noise transfer function without harmonic folding, while Part B corresponds to the harmonic folding. Similarly, linear analysis of  $v_{n,gm}(t)$  has the state-space description

$$\frac{\mathrm{d}v_{\mathrm{Ci}}(t)}{\mathrm{d}t} = \frac{v_{\mathrm{n,gm}}(t)}{\mathrm{C_i}\mathrm{R_1}} - \frac{v_{\mathrm{Ci}}(t)}{\mathrm{C_i}\mathrm{R_2}}$$
(26)

where

$$\begin{split} R_1 &= \frac{\alpha_3 + \frac{R_s}{R_L}}{\alpha_3 \beta_3 + \beta_3 \frac{R_s}{R_L} - \gamma_3 g_m R_s}, \ R_2 = \frac{\alpha_3 + \frac{R_s}{R_L}}{\alpha_3 \gamma_3} \\ \alpha_3 &= 1 + g_m R_s, \ \beta_3 = \frac{g_m}{\alpha_3} \left( \frac{R_s}{R_{F1}} + 1 \right) \\ \gamma_3 &= \frac{1}{R_L} + \frac{1}{R_{F1}} - \frac{g_m R_s}{\alpha_3 R_L} + \frac{R_s}{\alpha_3 R_L R_{F1}}. \end{split}$$

From (26) together with (16) and (17), the output noise PSD transfer function of  $G_{\rm m}$  stage from  $V_{\rm n,gm}$  to  $V_{\rm ci}$  [i.e.,  $H_{0,\rm gm}(j\omega)]$  and its harmonic folding [i.e.,  $H_{\rm n,gm}(j\omega)]$  can be derived. Finally, the output noise PSD to  $V_{\rm o}$  is

$$\overline{V_{n,\text{out,gm}}^{2}} = \frac{\left|V_{n,\text{gm}}(j\omega)\right|^{2} \left|g_{m} + H_{0,\text{gm}}g_{m} + \frac{H_{0,\text{gm}}}{R_{S}}\right|^{2}}{\left|\frac{1}{R_{s}} + \frac{1}{R_{L}} + g_{m}\right|^{2}} + \sum_{n=-\infty, n\neq 0}^{\infty} \left|g_{m} \frac{H_{n,\text{gm}}(j\omega)V_{n,\text{gm}}(j\omega - jn\omega_{s})}{\frac{1}{R_{s}} + \frac{1}{R_{L}} + g_{m}}\right|^{2}}{Part B}.$$
(27)

The simulated output noises at  $V_o$  due to  $v_{n,RS}(t)$  and  $v_{n,gm}(t)$ are shown in Fig. 10(a), whereas Fig. 10(b) and (c) show the output noise due to  $v_{n,sw}(t)$  and its key harmonic folding terms, respectively. Similar to the signal transfer function, the output noises from R<sub>S</sub> and G<sub>m</sub> are alike a comb, and can be considered as narrowband around  $n\omega_{\rm s}$ . Unlike the traditional wideband LNAs that have wideband output noise, here the output noise around the LO harmonics is much less than that at the LO first harmonic. Thus, a wideband passive mixer follows the GB-BPF for downconversion, with the noise due to harmonic folding being much relaxed. Besides, the noise transfer function of R<sub>sw</sub> is a notch function, while its harmonic folding terms are bandpass with much smaller amplitude. This is also true for the conventional N-path passive mixer as analyzed in [16, eq. 45] with a difference method. Around  $n\omega_s$  where the in-band signal exists, the main contribution to its noise is the folding from higher harmonics, which is much less than the OB noise. The noise from  $R_{\rm sw}$  is thus greatly suppressed, and a larger  $R_{\rm sw}$  is allowed to relax the LO power. In other words, by re-sizing g<sub>m</sub>, smaller switches can be used for the SC branch while keeping a high OB selectivity filtering profile as analyzed in Section II.

#### C. Intuitive Equivalent Circuit Model

As shown in Fig. 5(a) and (b), the filtering behavior at both  $V_{\rm i}$  and  $V_{\rm o}$  are similar to that of a single-ended passive mixer, which motivates the remodeling of the circuit in Fig. 1(a) with two sets of single-ended passive mixers: one at  $V_{\rm i}$  and one at  $V_{\rm o}$ , as shown in Fig. 11(a). With the proposed intuitive equivalent circuit, it is convenient to include the parasitic capacitances at

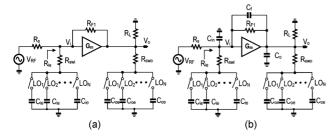


Fig. 11. Intuitive equivalent circuit of the GB-BPF. (a) Typical  $G_{\rm m}$  and (b) a non-ideal  $G_{\rm m}$  with parasitic capacitances  $C_{\rm in}$ ,  $C_{\rm o}$ , and  $C_{\rm f}$ .

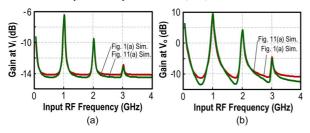


Fig. 12. Simulation comparison of Figs. 1(a) and 11(a). (a) Gain at  $V_i$  and (b) gain at  $V_o$ . The parameters are  $R_{\rm sw}=30~\Omega,~R_{\rm L}=80~\Omega,~R_{\rm S}=50~\Omega,~C_i=5~\rm pF,~g_m=100~mS,~R_{\rm F1}=500~\Omega,~f_{\rm Lo}=1~\rm GHz,$  and N=4.

both  $V_i$  and  $V_o$  by using a known theory developed in [14], [15] as shown in Fig. 11(b). The non-idealities due to LO phase/duty cycle mismatch can be analyzed similar to [14], while the variation of  $g_m$  to the in-band gain is similar to the condition of a simple inverter since the two sets of passive mixer are of high impedance at the clock frequency. Inside, we remodel the switch's on-resistance as  $R_{\rm swi}$  at  $V_i$  with capacitance  $C_{\rm ie}$ , and  $R_{\rm swo}$  at  $V_o$  with capacitance  $C_{\rm oe}$ .

$$\begin{cases} R_{swi} = \frac{(R_{sw}//R_{F1}) + R_L}{1 + g_m R_L} \approx \frac{R_{sw} + R_L}{1 + g_m R_L} \\ C_{ie} = \left| \frac{(1 - g_m R_{F1}) R_L}{R_L + R_{F1}} \right| \times C_i \\ R_{swo} = \frac{(R_{sw}//R_{F1}) + R_s}{1 + g_m R_s} \\ C_{oe} = C_i. \end{cases}$$
(28)

 $R_{\rm swi}$  described in (28) equals to (2). Thus, for far-out blockers,  $R_{\rm swi}//R_{\rm ie}$  is smaller than  $R_{\rm i}$ , which results in better ultimate rejection [Fig. 11(a)]. The value of  $C_{\rm ie}$  is obvious, it equals the gain of the circuit multiplied by  $C_{\rm i}$ , but without the SC branch in the feedback. It can be designated as the open-SC gain, and it can be enlarged to save area for a specific -3-dB bandwidth. As an example, with  $R_L=80~\Omega,~R_{\rm sw}=30~\Omega,~R_{\rm S}=50~\Omega,~C_{\rm i}=5~pF,~g_{\rm m}=100~mS$  and  $R_{\rm F1}=500~\Omega,~C_{\rm ie}$  is calculated to be 33.79 pF, which is  $\sim6\times$  smaller than  $C_{\rm i}$  in the traditional design [10], thus the area saving in  $C_{\rm i}$  is significant. For  $R_{\rm swo}$ , it equals the output resistance with  $R_{\rm sw}$  in the feedback. This is an approximated model without considering the loading from  $R_{\rm swi}$  to  $R_{\rm swo}$ .

To verify it, the frequency responses of Fig. 1(a) and Fig. 11(a) are plotted together in Fig. 12(a) and (b) for comparison. It is observed that their -3-dB bandwidth and gain around  $\omega_s$  fit well with each other, since the loading from the mutual coupling between the SC for IB signal is less an issue than that of OB blockers. As expected, the ultimate rejection in Fig. 11(a) is better than that in Fig. 1(a). Note that the parasitic capacitances  $C_{\rm in}$  at  $V_{\rm i}$  and  $C_{\rm o}$  at  $V_{\rm o}$  have been included in Fig. 11(b). Also, to account  $C_{\rm gs}$  of the  $G_{\rm m}$ 's two MOSFETs [Fig. 1(a)], a parasitic capacitance  $C_{\rm f}$  is placed in parallel with

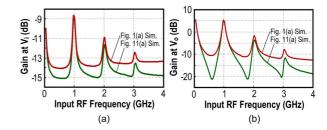


Fig. 13. Simulation comparison of Figs. 1(a) and 11(b). (a) Gain at  $V_{\rm i}$  and (b) gain at  $V_{\rm o}$ . The parameters are the same as Fig. 12, with the additional  $C_{\rm in}=1$  pF,  $C_{\rm o}=1$  pF, and  $C_{\rm f}=500$  fF.

### TABLE I KEY PARAMETERS IN THE DESIGN EXAMPLE

g <sub>m</sub> (mS)	$R_{sw}(\Omega)$	$R_{F1}(\Omega)$	$R_L(\Omega)$	C <sub>i</sub> (pF)
76	20	1k	120	5

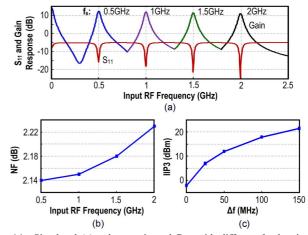


Fig. 14. Simulated (a) voltage gain and  $\rm S_{11}$  with different  $\rm f_s$  showing the LO-defined bandpass responses. (b) NF versus input RF frequency. (c) IB and OB IIP3.

 $R_{F1}.$  Still, the accuracy of the equivalent circuit is acceptable around  $f_{\rm s},$  as shown in Fig. 13(a) and (b). It is noteworthy that the gain at around  $\omega_{\rm s}$  fits better with each other than that of  $2\omega_{\rm s},3\omega_{\rm s},$  etc. For the influence of  $C_{\rm in}$  and  $C_{\rm o},$  it mainly lowers the IB gain and slightly shifts the resonant frequency [4], [14]. For  $C_{\rm f},$  it induces Miller equivalent capacitances at  $V_{\rm i}$  and  $V_{\rm o},$  further lowering the gain and shifting the center frequency. With (28) and the RLC model, the -3-dB bandwidth at  $V_{\rm i}$  is derived as

$$2\Delta f_{i3 \text{ dB}} = \frac{1}{4\pi \left(R_s / / \frac{R_{F1} + R_L}{1 + g_m R_L}\right) C_{ie}}$$

#### IV. DESIGN EXAMPLE

A four-path GB-BPF suitable for full-band mobile-TV or IEEE 802.11af cognitive radio is designed and simulated with 65-nm GP CMOS technology. The circuit parameters are summarized in Table I. The transistor sizes for the self-biased inverter-based  $G_{\rm m}$  are:  $(W/L)_{\rm PMOS}=(24/0.1)\times 4$  and  $(W/L)_{\rm NMOS}=(12/0.1)\times 4$ . The 0.1- $\mu m$  channel length is to raise the gain for a given power and  $g_{\rm m}$  value. The switches are NMOS with  $(W/L)_{\rm sw}=25/0.06$ .  $C_i$  is realized with MiM capacitor.

TABLE II
SIMULATED PERFORMANCE SUMMARY IN 65-nm CMOS

Tunable RF (GHz)	0.5 to 2
Gain (dB)	11 to 12.5
NF (dB)	2.14 to 2.23
IIP3 <sub>IB</sub> (dBm)*	-2
IIP3 <sub>OB</sub> (dBm) ( $\Delta f = +25 \text{ MHz}$ )*	+7
IIP3 <sub>OB</sub> (dBm) ( $\Delta f = +50 \text{ MHz}$ )*	+12
IIP3 <sub>OB</sub> (dBm) ( $\Delta f = +100 \text{ MHz}$ )*	+18
IIP3 <sub>OB</sub> (dBm) ( $\Delta f = +150 \text{ MHz}$ )*	+21.5
BW (MHz)	41 to 48
Power (mW) @ Supply (V)	7@1

 $^*f_s=500$  MHz, two tones at  $f_s+\Delta f+2$  MHz and  $f_s+2\Delta f+4$  MHz.

As shown in Fig. 14(a), the passband is LO-defined under  $f_{\rm s}=0.5,\,1,\,1.5,\,$  and 2 GHz and  $S_{11}<-15\,$  dB in all cases. The -3-dB BW ranges between 41 to 48 MHz, and is achieved with a total MiM capacitance of 20 pF. The calculated  $C_{\rm ie}$  based on (28) is thus  $\sim$ 40 pF, and the required  $C_{\rm ie}$  for four paths is 160 pF. The -3-dB BW at 2 GHz is larger due the parasitic capacitor that reduces the Q of the GB-BPF. The gain is 12.5 dB at 0.5-GHz RF, which drops to 11 dB at 2-GHz RF with an increase of NF by <0.1 dB as shown in Fig. 14(b). The IIP3 improves from IB (-2 dBm) to OB (+21.5 dBm at 150-MHz offset) as shown in Fig. 14(c). For the circuit non-idealities, 10% of LO duty cycle mismatch only induce a small variation of IB gain by around 0.05 dB. For a  $g_{\rm m}$  variation of 10%, the IB gain variation is 0.07 dB at 500-MHz LO frequency. The performance summary is given in Table II.

#### V. CONCLUSIONS

This paper has described the analysis, modeling and design of a GB-BPF that features a number of attractive properties. By using a transconductance amplifier (G<sub>m</sub>) as the forward path and an N-path SC branch as its feedback path, double RF filtering at the input and output ports of the G<sub>m</sub> is achieved concurrently. Moreover, when designed for input impedance matching, both in-band gain and bandwidth can be customized due to the flexibility created by G<sub>m</sub>. Both the power and area efficiencies are improved when compared with the traditional passive N-path filter due the loop gain offered by  $G_{\mathrm{m}}$ . All gain and bandwidth characteristics have been verified using a RLC model first, and later with the LPTV analysis to derive the R, L, and C expressions. The harmonic selectivity, harmonic folding and noise have been analyzed and verified by simulations, revealing that the noise of the switches is notched at the output, benefitting the use of small switches for the SC branch, saving the LO power without sacrificing the selectivity. The design example is a four-path GB-BPF. It shows > 11 dB gain, <2.3-dB NF over 0.5-2-GHz RF, and +21-dBm out-of-band IIP3 at 150-MHz offset, at just 7 mW of power. The developed models also backup the design of the ultra-low-power receiver in [9] for multiband sub-GHz ZigBee applications.

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