

# Hysteresis current control of hybrid active power filters

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**Abstract:** This article presents a hysteresis pulse width modulation study for a inductor-capacitor (LC)-coupling hybrid active power filter (HAPF). As the coupling LC impedance yields a non-linear inverter current slope, this can affect the controllability of using the conventional hysteresis control method and generate unexpected trigger signals to the switching devices. This results in deteriorating the system operating performances. On the basis of the proposed modelling, the linearisation of the hysteresis control for the HAPF is firstly studied, investigated and compared with the linear active power filter. Two limits are proposed in this study that divides the HAPF into non-linear, quasi-linear and linear operation regions. The design criteria of hysteresis band and sampling time can then be derived. Single-phase simulation and experimental results are given to verify the hysteresis control study of HAPF compared with active power filter. Finally, representative simulation and experimental results of a three-phase four-wire centre-split HAPF for power quality compensation are presented to demonstrate the validity of the hysteresis linearisation study.

## 1 Introduction

Pulse width modulation (PWM) techniques such as hysteresis, ramp comparison and space vector were traditionally developed based on linear concept [1]. They are usually applied on the conventional linear  $L$ -coupling voltage source inverters for different applications [1–9], such as ac motor drives, active power filter (APF) and dynamic voltage restorer. Among different PWM methods, hysteresis is one of the most popular PWM strategies [2–5] and widely applied in APF for current quality compensation, owing to its advantages such as ease of implementation, fast dynamic response and current limiting capability. However, the initial and operational costs of APF are high [10–12], thus slows down its large-scale application in distribution power networks. To overcome this problem, a transformerless LC-coupling hybrid active power filter (HAPF) has been recently proposed and applied for current quality compensation and damping of harmonic propagation in distribution power systems [13–16], in which it contains less passive components and the dc-link operating voltage can be much lower than the APF, thus reducing the system cost. Its low dc-link voltage characteristic is due to the system fundamental voltage dropped across the coupling capacitance but not the inverter part of the HAPF [14].

In recent research studies, the design principles and filtering characteristics for the LC-coupling HAPF have been presented and discussed [13]. An optimal design of its dc-link voltage has been introduced based on the vector trajectory method [14]. In order to enhance the LC-coupling HAPF system response, a novel control technique has also been proposed [15]. As the coupling LC yields a non-linear inverter current slope, this can affect the controllability of

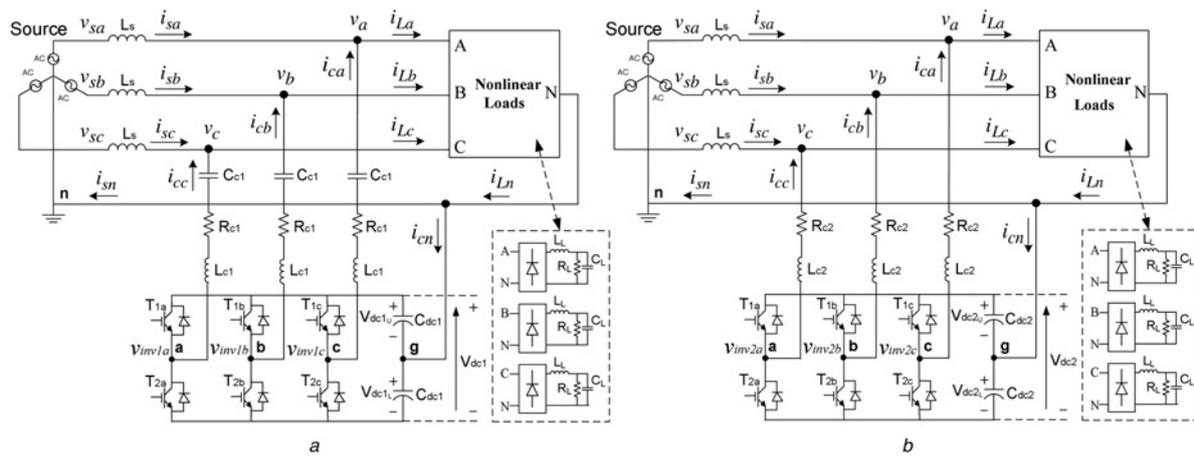
using the conventional hysteresis control method and generate unexpected trigger signals to the switching devices, thus deteriorating the system operating performances. However, there is still no research work examining nor discussing the minimum operating requirement for this HAPF (i.e. LC-coupling voltage source inverter) if the linear hysteresis control strategy is applied.

In this study, the modelling of the LC-coupling HAPF will be firstly presented. Via its modelling, the linearisation work of hysteresis control for this HAPF will be studied in comparison to the linear APF. Two limits quasi-linear limit and linear limit which divide the HAPF into non-linear, quasi-linear and linear operation regions are proposed. With respect to different LC values, this study aims to define the corresponding requirements for hysteresis band and sampling time, such that the inverter current slope of the HAPF can be treated as linear using the hysteresis method. Otherwise, unexpected trigger signals will be generated to the switching devices and strongly affect the HAPF compensating performances. Finally, simulation and experimental results for single-phase and three-phase systems are presented to verify the hysteresis control study for HAPF compared with that of linear APF.

## 2 Modelling of a two-level three-phase four-wire centre-split HAPF and APF

### 2.1 Circuit configuration of a two-level three-phase four-wire centre-split HAPF and APF

Fig. 1 illustrates the circuit configuration of a two-level three-phase four-wire HAPF and APF, where the subscript 'x' denotes phase  $a$ ,  $b$ ,  $c$ ,  $n$ .  $v_{sx}$  is the system voltage,  $v_x$  is the load voltage,  $L_s$  is the system inductance normally



**Fig. 1** Circuit configuration of a two-level three-phase four-wire centre-split compensator

a HAPF  
b APF

neglected due to its low value relatively, thus  $v_{sx} \approx v_x$ .  $i_{sx}$ ,  $i_{Lx}$  and  $i_{cx}$  are the system, load and inverter current for each phase.  $C_{c1}$ ,  $L_{c1}$  and  $R_{c1}$  are the coupling part capacitance, inductance and internal resistance of the HAPF, in which  $C_{c1}$  and  $L_{c1}$  are designed basing on the reactive power consumption and the dominant harmonic current of the inductive loading.  $L_{c2}$  and  $R_{c2}$  are the coupling part inductance and internal resistance of the APF, in which  $L_{c2}$  is designed basing on the switching frequency, switching ripple and dc-link voltage-level consideration.  $C_{dc1}$ ,  $V_{dc1U}$  and  $V_{dc1L}$  are the dc capacitor, upper and lower dc capacitor voltages of the HAPF, while  $C_{dc2}$ ,  $V_{dc2U}$  and  $V_{dc2L}$  are the dc capacitor, upper and lower dc capacitor voltages of the APF. The dc-link midpoint is assumed to be ground reference (g). From Fig. 1, the inverter line-to-ground voltages  $v_{invjx-g}$  will be equal to the inverter line-to-neutral voltages  $v_{invjx-n}$  because the neutral point n is connected with the dc-link midpoint g, where subscript  $j=1$  represents the HAPF and  $j=2$  represents the APF. The non-linear loads are composed of three single-phase diode bridge rectifiers, which act as harmonic producing loads in this study. In the following, all the analyses are based on sufficient dc-link voltage assumption.

### 2.2 Modelling of a two-level three-phase four-wire centre-split HAPF and APF

From Fig. 1, the switching functions  $S_x$  for each inverter leg of HAPF and APF can be given as

$$S_x = \begin{cases} 1 & \text{when } T_{1x} \text{ is ON and } T_{2x} \text{ is OFF} \\ -1 & \text{when } T_{1x} \text{ is ON and } T_{2x} \text{ is OFF} \end{cases} \quad (1)$$

For  $V_{dcjU} = V_{dcjL} = 0.5 V_{dcj}$ , the output voltage for each inverter leg can be expressed as

$$v_{invjx} = S_x \times 0.5 V_{dcj} \quad (2)$$

The inverter line-to-neutral voltage  $v_{invjx-n}$  can be expressed as

$$v_{invjx-n} = v_{invjx} - v_n = v_{invjx} \quad (3)$$

where the neutral line voltage  $v_n = 0$ , the switching state of an

inverter leg is independent of the switching states of the other two legs, the inverter line-to-neutral voltage  $v_{invjx-n}$  for each leg is either equal to  $-0.5V_{dcj}$  or  $+0.5V_{dcj}$ . From Fig. 1a, the relationship among the inverter current  $i_{cx}$ , load voltage  $v_x$ , coupling capacitor voltage  $v_{Cc1x}$  and inverter line-to-neutral voltage  $v_{inv1x}$  of the HAPF is

$$L_{c1} \frac{di_{cx}(t)}{dt} + R_{c1}i_{cx}(t) = v_{inv1x}(t) - v_x(t) - v_{Cc1x}(t) \quad (4)$$

From (2), the modelling for each inverter leg of the HAPF can be expressed as

$$L_{c1} \frac{di_{cx}(t)}{dt} + R_{c1}i_{cx}(t) = S_x \times 0.5 V_{dc1} - v_x(t) - v_{Cc1x}(t) \quad (5)$$

$$C_{c1} \frac{dv_{Cc1x}(t)}{dt} = i_{cx}(t) \quad (6)$$

From Fig. 1b, the relationship among the inverter current  $i_{cx}$ , load voltage  $v_x$  and inverter line-to-neutral voltage  $v_{inv2x}$  of the APF is

$$L_{c2} \frac{di_{cx}(t)}{dt} + R_{c2}i_{cx}(t) = v_{inv2x}(t) - v_x(t) \quad (7)$$

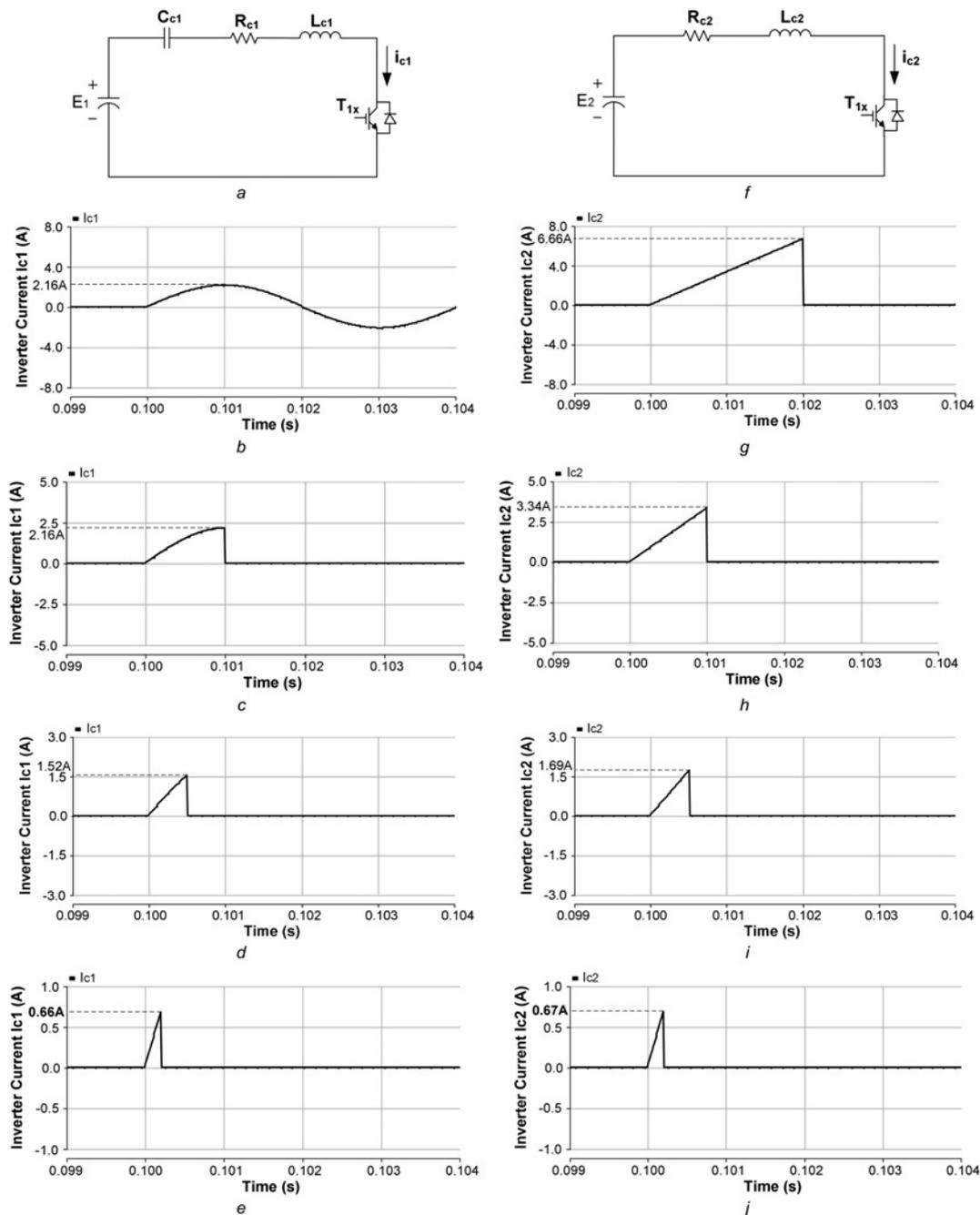
From (2), the modelling for each inverter leg of the APF can be expressed as

$$L_{c2} \frac{di_{cx}(t)}{dt} + R_{c2}i_{cx}(t) = S_x \times 0.5 V_{dc2} - v_x(t) \quad (8)$$

Via (4)–(8), the linearisation study of hysteresis control for the HAPF in comparison to the linear APF will be presented in the following.

### 3 Hysteresis PWM control for the HAPF

As the coupling LC of the HAPF yields a non-linear inverter current slope, this can affect the controllability of using the conventional hysteresis PWM, the linearisation of the hysteresis control for the HAPF should be investigated. In this study, the slope of the HAPF inverter current  $i_{cx}$  is categorised into three operation regions: (i) non-linear



**Fig. 2** Simulated  $i_{c1}$  of HAPF and  $i_{c2}$  of APF single-phase model at different  $t_{on}$  interval and  $E_1, E_2 = 20 V$

HAPF:

a Single-phase model for  $T_{1x}$  at different  $t_{on}$  interval

b  $t_{on} = 2000 \mu s$  (non-linear)

c  $t_{on} = 1000 \mu s$  (quasi-linear)

d  $t_{on} = 500 \mu s$  (quasi-linear)

e  $t_{on} = 200 \mu s$  (linear)

APF:

f Single-phase model for  $T_{1x}$  at different  $t_{on}$  interval

g  $t_{on} = 2000 \mu s$  (linear)

h  $t_{on} = 1000 \mu s$  (linear)

i  $t_{on} = 500 \mu s$  (linear)

j  $t_{on} = 200 \mu s$  (linear)

region, (ii) quasi-linear region and (iii) linear region, in which their corresponding definitions and diagrams under hysteresis control are given in Appendix 8.1. In addition, these three operation regions are separated by two limits: quasi-linear limit  $T_{limit}$  and linear limit  $T_{linear}$ , in which their values are determined based on a constant reference inverter current  $i_{cx}^*$

assumption as given in Appendix 8.2. After that, the corresponding quasi-linear limit  $H_{limit}$  and linear limit  $H_{linear}$  for the hysteresis band can be obtained in Appendix 8.3. In the following, these three definitions and two limits will be used for the HAPF inverter current slope linearity justification under hysteresis control method.

In practical case, since different loading situations may require different coupling  $C_{c1}$  and  $L_{c1}$  values, thus yielding different  $T_{limit}$  ( $H_{limit}$ ) and  $T_{linear}$  ( $H_{linear}$ ) values (Appendix 8.1–8.3), it is important and necessary to carry out this linearity analysis, so that the linear operation region for each HAPF system can be achieved. In order to simplify the verification, the coupling part internal resistance for the HAPF and APF is approximately treated as zero ( $R_{c1} \simeq 0$  and  $R_{c2} \simeq 0$ ) in the following simulations and experiments because their values are usually small in practical situation.

### 3.1 Single-phase simulation and experimental results for HAPF inverter current slope linearisation analysis

Under constant reference inverter current  $i_{cx}^*$  assumption, Figs. 2a and f illustrate the HAPF and APF single-phase inverter current slope linearisation study models for upper switch  $T_{1x}$  at different switching on interval ( $t = t_{on}$ ), in which the upper switch is an insulated gate bipolar transistor (IGBT).  $E_1$  and  $E_2$  represent the instantaneous peak voltage difference between the inverter output voltage and load voltage. Reversing the polarity of  $E_1$  and  $E_2$  will yield their models for lower switch  $T_{2x}$  ( $t = t_{off}$ ). As the only difference between upper and lower switches is the opposite current polarity, the results for  $T_{2x}$  will not be included in this study. In short,  $E_1$  and  $E_2$  can be treated as dc voltage inputs. In addition, the load voltage is treated as a constant value for the following analysis. Table 1 lists the system parameters of HAPF and APF single-phase models. When  $R_{c1} \simeq 0$  and  $R_{c2} \simeq 0$ , the current slope for APF will be linear no matter what the switching interval is, as verified by (7) and (8). As  $E_1$  and  $E_2$  values do not affect the determination results of the quasi-linear limit and linear limit,  $E_1$  and  $E_2$  are chosen as 20 V in this study. From Table 1, the quasi-linear limit and linear limit for HAPF will be  $T_{limit} = 1000 \mu s$  and  $T_{linear} = 200 \mu s$ , respectively. Simulation studies were carried out by using power system computer aided design (PSCAD)/electro magnetic transient in DC system (EMTDC). In the following, simulation and experimental results for Figs. 2a and f will be given to verify the HAPF inverter current slope linearisation analysis compared with the linear APF. Figs. 2 and 3 illustrate the simulated and experimental inverter current  $i_{c1}$  and  $i_{c2}$  for their single-phase models at different switching on interval.

In order to satisfy the linear region definition  $|(di_{cx})/dt|_{error} \leq 5\%$  (Appendix 8.1–8.2), the slope error of HAPF inverter current  $i_{c1}$  compared with the APF one  $i_{c2}$  should be less than 5%. From Fig. 2b, when  $t_{on} > 1000 \mu s$ , which is larger than the quasi-linear limit ( $T_{limit} = 1000 \mu s$ ), the slope of  $i_{c1}$  as in Fig. 2b is non-linear compared with that of  $i_{c2}$  as shown in Fig. 2g. However, even when  $t_{on} = 1000$  and  $500 \mu s$  satisfy the quasi-linear limit, they do not satisfy the linear limit. It is obvious that Figs. 2c–d contain

**Table 1** System parameters of HAPF and APF single-phase models

| System parameters | Physical values   |
|-------------------|-------------------|
| $L_{c1}, L_{c2}$  | 6 mH              |
| $R_{c1}, R_{c2}$  | $\simeq 0 \Omega$ |
| $C_{c1}$          | 70 $\mu F$        |
| $E_1, E_2$        | 20 V (dc)         |

current slope errors (35.3%, 10.0%) compared with that of APF as shown in Figs. 2h–i. These errors have the potential of deteriorating the system operating performance significantly. When  $t_{on} = 200 \mu s$ , which satisfies the linear limit ( $T_{linear} = 200 \mu s$ ), the current slope error decreases to 1.5% (within the linear region definition  $|(di_{cx})/dt|_{error} \leq 5\%$ ),  $i_{c1}$  in Fig. 2e is approximately the same as  $i_{c2}$  in Fig. 2j.

The experimental current  $i_{c1}$  and  $i_{c2}$  for their single-phase models at different switching on interval ( $t = t_{on}$ ) are shown in Fig. 3, in which the experimental results are consistent with the simulation results as shown in Fig. 2. When  $t_{on} > 1000 \mu s$ , the slope of  $i_{c1}$  as shown in Fig. 3a is non-linear compared with that of  $i_{c2}$  as shown in Fig. 3e. Even when  $t_{on} = 1000$  and  $500 \mu s$  satisfy the quasi-linear limit ( $T_{limit} = 1000 \mu s$ ), it is obvious that Figs. 3b and 3c contain current slope errors (35.1%, 13.4%) compared with that of APF as in Figs. 3f and g. When  $t_{on} = 200 \mu s$ , which satisfies the linear limit ( $T_{linear} = 200 \mu s$ ), the current slope error decreases to 4.6% (within the linear region definition),  $i_{c1}$  in Fig. 3d is approximately the same as  $i_{c2}$  in Fig. 3h.

Since the load voltage is treated as a constant value in the analysis, its inconstant effect to the HAPF non-linearity analysis can be neglected. From Figs. 2 and 3, APF always obtains a linear current slope, whereas HAPF can obtain a non-linear current slope due to its inherent non-linear characteristic. The single-phase simulation and experimental results verified the hysteresis linearisation study of HAPF compared with that of linear APF.

### 3.2 Summary for determining quasi-linear limit $T_{limit}$ and linear limit $T_{linear}$

In summary, the quasi-linear limit  $T_{limit}$ , linear limit  $T_{linear}$  and their corresponding hysteresis bands ( $H_{limit}, H_{linear}$ ) for the HAPF can be determined in the following steps:

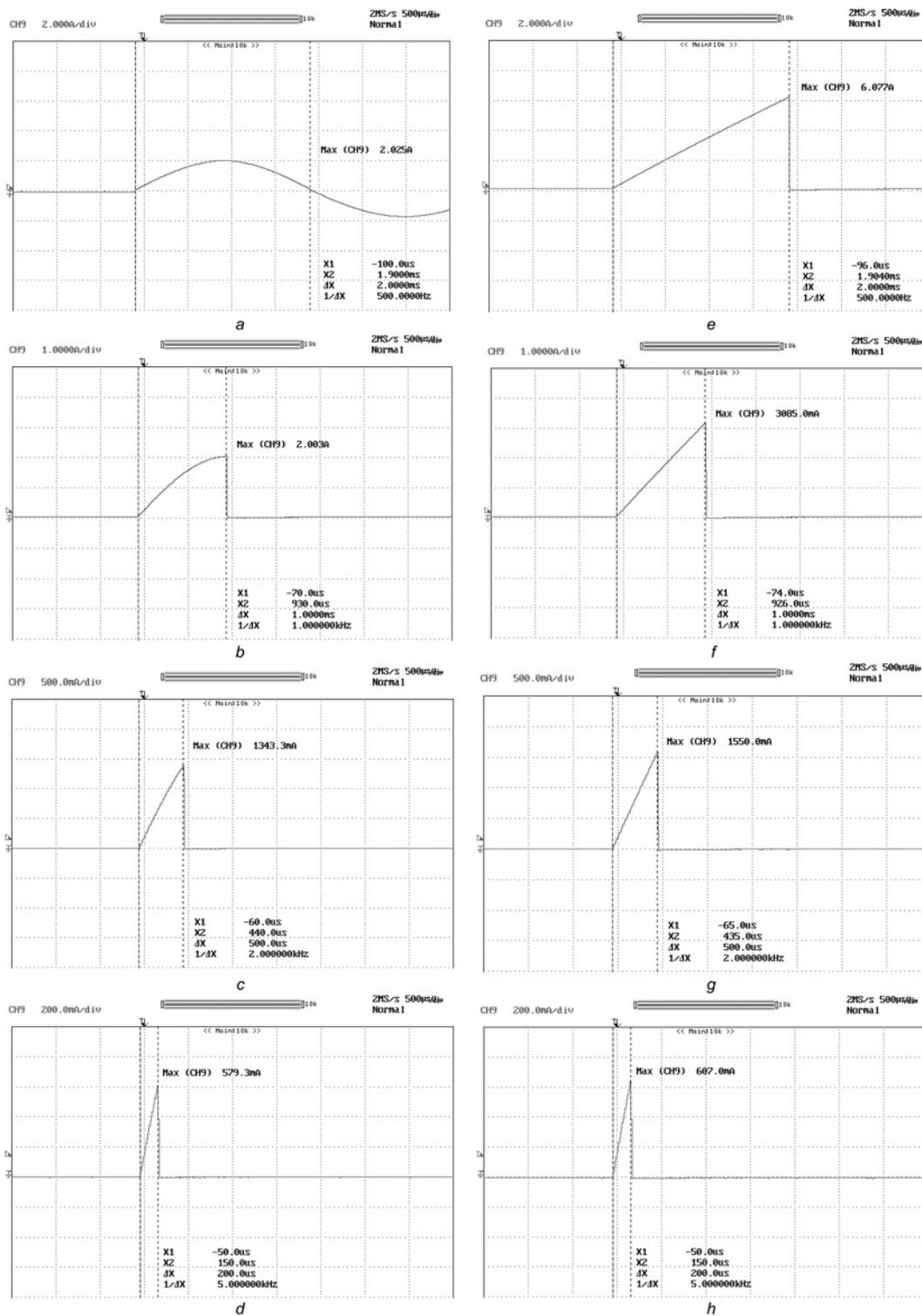
1. Input system parameters  $L_{c1}$  and  $C_{c1}$ , by setting  $A = \cos \omega_r t = 0$ ,  $T_{limit}$  is the minimum value among all  $t$  values when  $A = 0$  (excluding  $t = 0$ ), where  $\omega_r = (1/\sqrt{L_{c1}C_{c1}})$ .  $H_{limit}$  can then be obtained by (14).
2. Finding  $T_{linear}$  value by setting  $|(di_{cx})/dt|_{error} = |1 - \cos \omega_r t| = \varepsilon\%$  (11),  $\varepsilon$  is set to 5% in this study (Appendix 8.2).  $H_{linear}$  can then be obtained by (14).

## 4 HAPF linearisation study verification under three-phase four-wire power quality compensator application

The previous linearisation analyses and verifications are based on constant reference inverter current  $i_{cx}^*$  assumption (Appendix 8.2), which yields a fixed switching frequency as verified by (14) and illustrated in Fig. 4a. However,  $i_{cx}^*$  is not a constant value and unpredictable in practical three-phase four-wire power quality compensator application, thus the switching frequency of the practical HAPF system under a fixed hysteresis band value is not fixed as verified by (13) and illustrated in Fig. 4b. From Fig. 4b, when the slope of the inverter current  $i_{cx}$  keeps the same as Fig. 4a, it is obvious that  $t_{on1}, t_{on2} > T_{linear}$  and  $t_{off1}, t_{off2} < T_{linear}$  due to the variation of reference inverter current  $i_{cx}^*$ . In addition, when the slope of  $i_{cx}$  is the same polarity as the reference  $i_{cx}^*$ ,  $t_{on1}, t_{on2} > T_{linear}$  and vice versa. Owing to  $H_{linear}$  is set

under a small slope error of  $\varepsilon = 5\%$  in this study, it can also be treated as the linear hysteresis band limit during practical case.

In practical three-phase four-wire power quality compensator application, by setting the hysteresis band  $H$  significantly smaller than the quasi-linear limit  $H_{limit}$  or linear limit  $H_{linear}$ , it is



**Fig. 3** Experimental  $i_{c1}$  of HAPF and  $i_{c2}$  of APF single-phase model at different  $t_{on}$  interval and  $E_1, E_2 = 20 V$

HAPF:

- a  $t_{on} = 2000 \mu s$  (non-linear)
- b  $t_{on} = 1000 \mu s$  (quasi-linear)
- c  $t_{on} = 500 \mu s$  (quasi-linear)
- d  $t_{on} = 200 \mu s$  (linear)

APF:

- e  $t_{on} = 2000 \mu s$  (linear)
- f  $t_{on} = 1000 \mu s$  (linear)
- g  $t_{on} = 500 \mu s$  (linear)
- h  $t_{on} = 200 \mu s$  (linear)

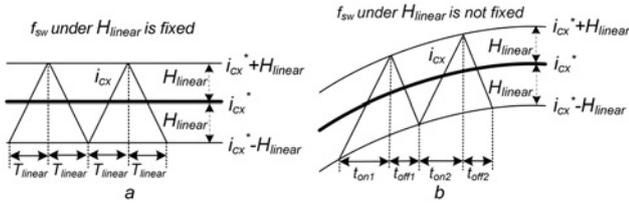


Fig. 4 Hysteresis PWM control under  $H_{linear}$  based on

- a Previous constant  $i_{cx}^*$  assumption
- b Inconstant  $i_{cx}^*$  consideration

Table 2 HAPF simulated and experimental system parameters for power quality compensation

| System parameters            | Physical values                       |
|------------------------------|---------------------------------------|
| $v_x$ ( $x = a, b, c$ ), $f$ | 55 $V_{rms}$ , 50 Hz                  |
| $L_s$                        | 0.5 mH                                |
| $L_{c1}, C_{c1}, R_{c1}$     | 6 mH, 70 $\mu F$ , $\approx 0 \Omega$ |
| $C_{dc1}$                    | 5 mF                                  |
| $V_{dc1U}, V_{dc1L}$         | 30 V                                  |
| $L_L, C_L, R_L$              | 30 mH, 202 $\mu F$ , 25 $\Omega$      |

possible to keep the HAPF operates in quasi-linear region or linear region.

#### 4.1 Determination of final sampling time $T_{final}$ and hysteresis band $H_{final}$

The final sampling time  $T_{final}$  and hysteresis band  $H_{final}$  for the HAPF can be determined in the following steps:

1. For the fastest sampling time by AD converter is  $T_{fastest}$ , the final sampling time  $T_{final}$  can be chosen as  $T_{fastest} \leq T_{final} \leq T_{linear}$ .
2. For the switching frequency limitation of the switching devices is  $f_{swlimit(SD)}$ , the hysteresis band  $H_{swlimit(SD)}$  based on  $f_{swlimit(SD)}$  can be obtained through (14). Then the final hysteresis band can be chosen as  $H_{swlimit(SD)} \leq H_{final} \leq H_{linear}$  in order to obtain a linear hysteresis control for the HAPF.

In the following, the HAPF linearisation study under three-phase four-wire power quality compensator application as shown in Fig. 1a will be investigated by both simulations and experiments. In order to simplify the verification in this part, the dc link is supported by external dc voltage source and the simulated and experimental three-phase loadings are approximately balanced. Simulation studies were carried out by using PSCAD/EMTDC. In order to verify the simulation results, a three-phase four-wire centre-split HAPF experimental prototype is also implemented in the laboratory. The control system of the prototype is a digital signal processor (DSP) TMS320F2407 and the sampling time is chosen as 50  $\mu s$  (20 kHz). Moreover, the Mitsubishi IGBT intelligent power modules PM300DSA060 are employed as the switching devices for the inverter. The switching frequency limitation of the switching devices  $f_{swlimit(SD)}$  is 20 kHz and the corresponding  $H_{swlimit(SD)} = 0.0625$  A. Table 2 lists the HAPF simulated and experimental system parameters for power quality compensation. From Table 2, the quasi-linear limit and linear limit for HAPF can be calculated as  $T_{limit} = 1000 \mu s$

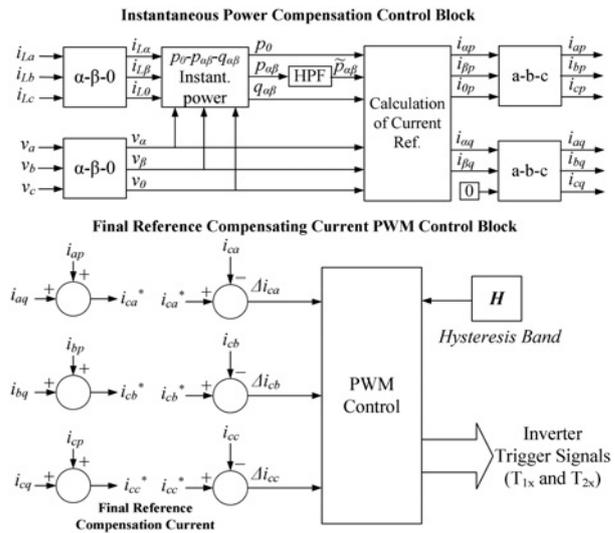


Fig. 5 Control block diagram for the three-phase four-wire centre-split HAPF

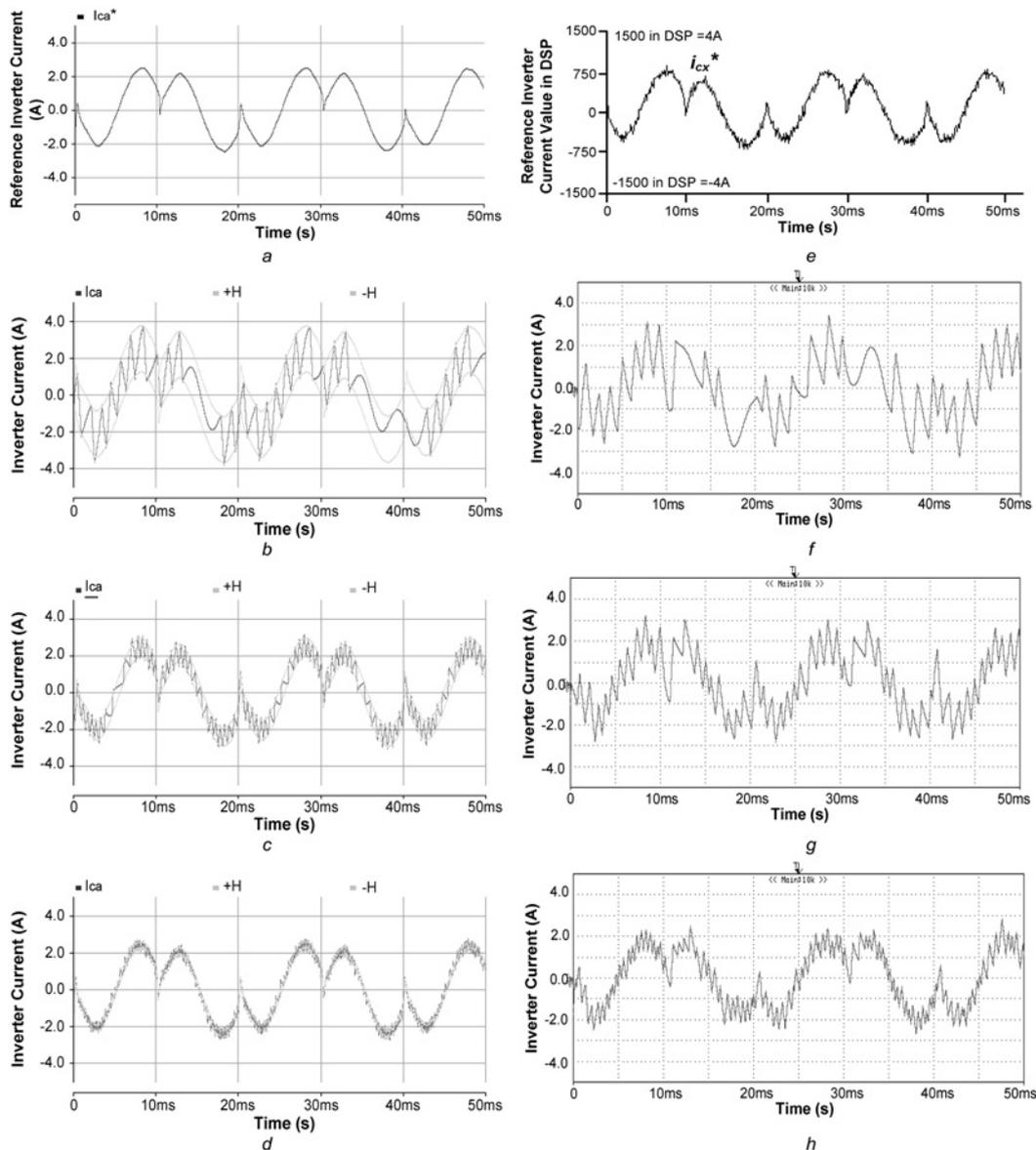
( $H_{limit} = 1.25$  A) and  $T_{linear} = 200 \mu s$  ( $H_{linear} = 0.25$  A), respectively.

Fig. 5 shows the control block diagram for the three-phase four-wire HAPF, in which it consists of two main control blocks: instantaneous power compensation control block and final reference compensating current PWM control block. From Fig. 5, the reference compensating active and reactive currents for the HAPF ( $i_{xp}, i_{xq}$ , the subscript  $x = a, b, c$  for three-phases) are determined based on the three-phase instantaneous  $pq$  theory [17]. The final reference compensating current  $i_{cx}^*$  can be obtained by summing up the  $i_{xp}$  and  $i_{xq}$ , which can compensate both the reactive, harmonic and neutral currents. Then the compensating current error  $\Delta i_{cx}$  together with hysteresis band  $H$  will be sent to the current PWM control part for generating PWM trigger signals to the IGBTs. In the following, representative simulation and experimental results are included to illustrate and verify the HAPF inverter current slope linearisation study under three-phase power quality compensator application.

#### 4.2 Simulation results

Fig. 6a shows the simulated reference inverter current  $i_{cx}^*$ , and Figs. 6b–d depict the simulated inverter current  $i_{cx}$  when the HAPF is operating at non-linear region, quasi-linear region and linear region. Table 3 summarises their corresponding simulation results before and after compensation.

1. *Non-linear region:* As  $T_{limit} = 1000 \mu s$  ( $H_{limit} = 1.25$  A), the hysteresis band  $H = 1.25$  A will keep the HAPF operating at non-linear region. The polarity of the inverter current slope has interchanging before  $i_{cx}$  touches the band  $H$  at some instances, which can be verified by Fig. 6b. Table 3 illustrates that the HAPF can compensate the system displacement power factor (DPF) from 0.83 into unity. However, the non-linear slope of  $i_{cx}$  deteriorates the compensation results ( $THD_{i_{sx}} = 31.2\%$ ,  $THD_{V_x} = 7.2\%$ , system neutral current  $i_{sn} = 1.40 A_{rms}$ ) by generating unexpected trigger signals to the switching devices. Moreover, both compensated  $THD_{i_{sx}}$  and  $THD_{V_x}$  do not satisfy the international standards [18, 19]. From Table 3, the system average operating switching frequency is 0.6 kHz.



**Fig. 6** Simulated and experimental  $i_{cx}^*$  and  $i_{cx}$  for HAPF at different operation regions

Simulated results:

- a Reference inverter current  $i_{cx}^*$
- b Non-linear region ( $H = 1.25$  A)
- c Quasi-linear region ( $H = 0.50$  A)
- d Linear region ( $H = 0.156$  A)

Experimental results:

- e Reference inverter current  $i_{cx}^*$  in DSP
- f Non-linear region ( $H = 1.25$  A)
- g Quasi-linear region ( $H = 0.50$  A)
- h Linear region ( $H = 0.156$  A)

2. *Quasi-linear region*: Setting the band  $H = 0.50$  A can keep the HAPF operating at quasi-linear region. The polarity of the inverter current slope changes only when  $i_{cx}$  touches the band  $H$ , as shown in Fig. 6c. The system DPF has been compensated into unity and  $i_{sn} = 0.68 A_{rms}$ . When HAPF is operating at quasi-linear region, the current slope error is significant, which affects the system performances. This phenomenon can be verified by showing compensated  $THD_{i_{sx}} = 17.3\%$  and  $THD_{V_x} = 6.6\%$  as shown in Table 3, in which both of them do not satisfy the international standards [18, 19]. From Table 3, the system average operating switching frequency is 1.7 kHz.

3. *Linear region*: As  $T_{linear} = 200 \mu s$  ( $H_{linear} = 0.25$  A) and  $H_{swlimit(SD)} = 0.0625$  A, the hysteresis band  $H = 0.156$  A can satisfy both the linear limit and the switching frequency limitation of the switching devices. This band value can keep the HAPF operating at linear region as verified by Fig. 6d. Table 3 illustrates that  $H = 0.156$  A yields the best compensation performances among the three operation regions. The system current, system neutral current, system DPF after compensation becomes sinusoidal ( $THD_{i_{sx}} = 6.2\%$ ), less ( $0.32 A_{rms}$ ), unity and  $THD_{V_x} = 2.8\%$ , in which all the results satisfy the international standards [18, 19]. From Table 3, the system

**Table 3** Simulation results for hysteresis PWM controlled HAPF before and after compensation at different operation regions

|   | HAPF with $V_{dc1U}, V_{dc1L} = 30\text{ V}, C_{c1} = 70\ \mu\text{F}, L_{c1} = 6\text{ mH}$ |   |   |                                      |
|---|--|---|---|--------------------------------------|
|   | Before compensation  | After compensation                      |   |                                      |
|   |  | (a) $H = 1.25\text{ A}$<br>(non-linear) | (b) $H = 0.50\text{ A}$<br>(quasi-linear) | (c) $H = 0.156\text{ A}$<br>(linear) |
| system current $i_{sx}$ ( $A_{rms}$ )                         | 2.67   | 2.36                                    | 2.17                                      | 2.12                                 |
| system DPF  | 0.83   | 1.00                                    | 1.00                                      | 1.00                                 |
| total harmonics distortion of system current THD $_{isx}$ (%) | 28.0   | 31.2                                    | 17.3                                      | 6.2                                  |
| total harmonics distortion of load voltage THD $_{Vx}$ (%)    | 2.1  | 7.2                                     | 6.6                                       | 2.8                                  |
| system neutral current $i_{sn}$ ( $A_{rms}$ )                 | 2.18   | 1.40                                    | 0.68                                      | 0.32                                 |
| average operating switching frequency $f_{sw}$ (Hz)           | –  | 0.6 k                                   | 1.7 k                                     | 5.1 k                                |

average operating switching frequency is 5.1 kHz. When  $H$  is further decreased, the HAPF can obtain better compensation results as the slope error can be reduced.

### 4.3 Experimental results

Fig. 6e shows the experimental reference inverter current  $i_{cx}^*$  in DSP, and Figs. 6f–h depict the experimental inverter current  $i_{cx}$  when the HAPF is operating at non-linear region, quasi-linear region and linear region. Table 4 summarises their experimental results before and after compensation.

1. *Non-linear region:* As  $T_{limit} = 1000\ \mu\text{s}$  ( $H_{limit} = 1.25\text{ A}$ ), the hysteresis band  $H = 1.25\text{ A}$  will keep the HAPF operating at non-linear region. The polarity of the inverter current slope has interchanging before  $i_{cx}$  touches the band  $H$  at some instances, as shown in Fig. 6f. Table 4 illustrates that the HAPF can compensate the system DPF close to unity. However, the non-linear slope of  $i_{cx}$  deteriorates the compensation results (THD $_{isx} = 27.3\%$ , THD $_{Vx} = 5.6\%$  for the worst phase,  $i_{sn} = 1.46\text{ A}_{rms}$ ) by generating unexpected

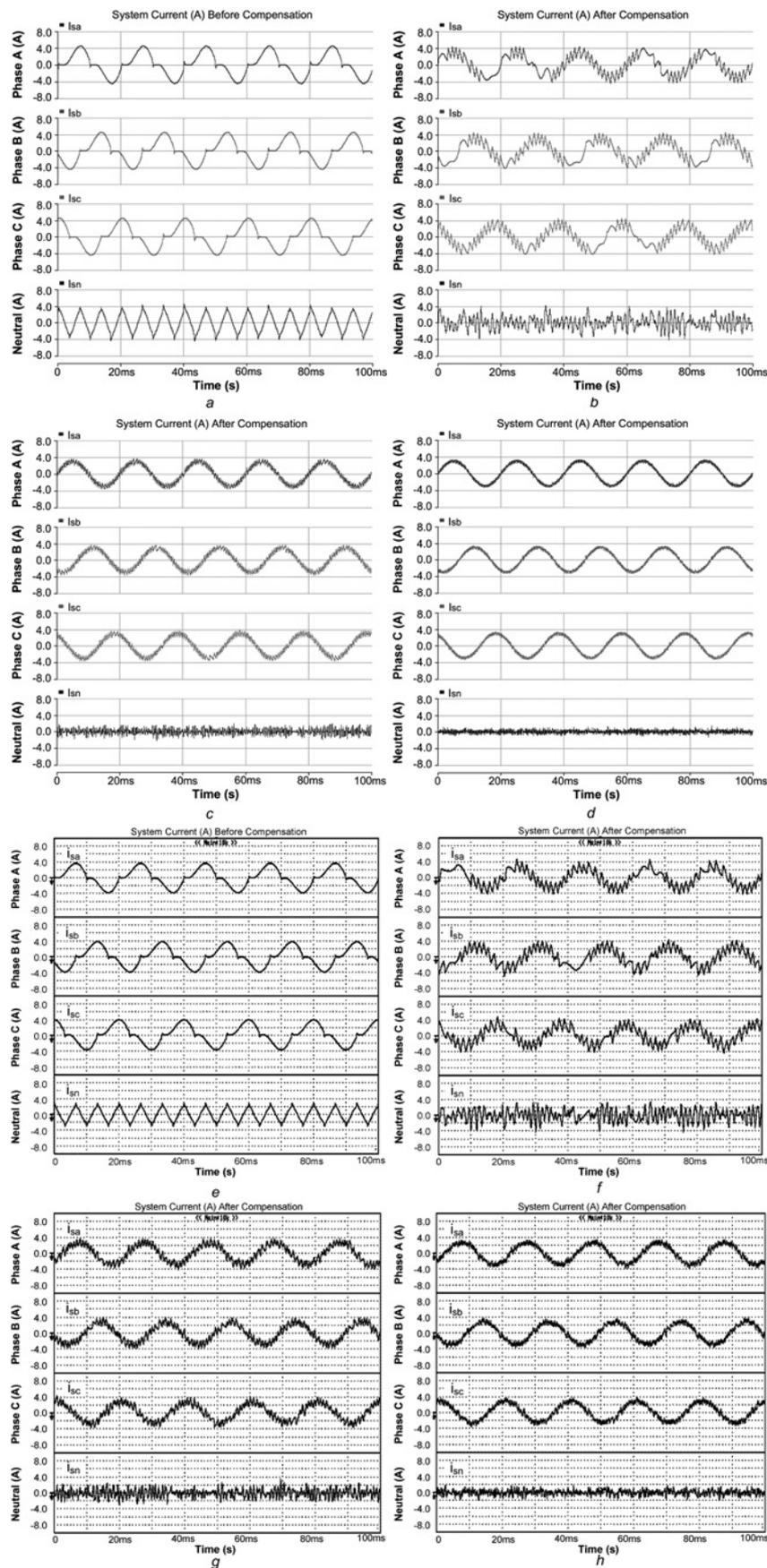
trigger signals to the switching devices. Moreover, both compensated THD $_{isx}$  and THD $_{Vx}$  do not satisfy the international standards [18, 19]. From Table 4, the system average operating switching frequency is 0.5 kHz.

2. *Quasi-linear region:* Setting the band  $H = 0.50\text{ A}$  can keep the HAPF operating at quasi-linear region. The polarity of the inverter current slope changes only when  $i_{cx}$  touches the band  $H$ , as shown in Fig. 6g. The system DPF has been compensated close to unity. When HAPF is operating at quasi-linear region, the current slope error is significant, which affects the operating performances (THD $_{isx} = 16.4\%$  and THD $_{Vx} = 4.3\%$  for the worst phase,  $i_{sn} = 0.98\text{ A}_{rms}$ ) as shown in Table 4, and the compensated THD $_{isx}$  does not satisfy the international standards [18, 19]. From Table 4, the system average operating switching frequency is 1.1 kHz.

3. *Linear region:* As  $T_{linear} = 200\ \mu\text{s}$  ( $H_{linear} = 0.25\text{ A}$ ) and  $H_{swlimit(SD)} = 0.0625\text{ A}$ , the hysteresis band  $H = 0.156\text{ A}$  can satisfy both the linear limit and the switching frequency limitation of the switching devices. This band value can keep the HAPF operating at linear region as verified by

**Table 4** Experimental results for hysteresis PWM controlled HAPF before and after compensation at different operation regions

|   | HAPF with $V_{dc1U}, V_{dc1L} = 30\text{ V}, C_{c1} = 70\ \mu\text{F}, L_{c1} = 6\text{ mH}$ |                     |   |   |                                      |
|---|--|---------------------|---|---|--------------------------------------|
|   |  | Before compensation | After compensation                      |   |                                      |
|   |  |                     | (a) $H = 1.25\text{ A}$<br>(non-linear) | (b) $H = 0.50\text{ A}$<br>(quasi-linear) | (c) $H = 0.156\text{ A}$<br>(linear) |
| system current $i_{sx}$ ( $A_{rms}$ )                         | A  | 2.42                | 2.21                                    | 2.13                                      | 2.10                                 |
|   | B  | 2.42                | 2.20                                    | 2.14                                      | 2.11                                 |
|   | C  | 2.35                | 2.07                                    | 1.96                                      | 1.94                                 |
| system DPF  | A  | 0.86                | 1.00                                    | 1.00                                      | 1.00                                 |
|   | B  | 0.89                | 1.00                                    | 1.00                                      | 1.00                                 |
|   | C  | 0.86                | 1.00                                    | 1.00                                      | 1.00                                 |
| total harmonics distortion of system current THD $_{isx}$ (%) | A  | 23.2                | 27.3                                    | 16.1                                      | 9.5                                  |
|   | B  | 22.4                | 25.9                                    | 16.4                                      | 9.2                                  |
|   | C  | 23.4                | 26.2                                    | 16.0                                      | 9.8                                  |
| total harmonics distortion of load voltage THD $_{Vx}$ (%)    | A  | 2.5                 | 5.6                                     | 4.3                                       | 2.8                                  |
|   | B  | 2.4                 | 5.2                                     | 4.2                                       | 3.0                                  |
|   | C  | 2.5                 | 5.1                                     | 4.2                                       | 2.9                                  |
| system neutral current $i_{sn}$ ( $A_{rms}$ )                 | N  | 1.57                | 1.46                                    | 0.98                                      | 0.64                                 |
| average operating switching frequency $f_{sw}$ (Hz)           | A  | –                   | 0.5 k                                   | 1.1 k                                     | 3.1 k                                |
|   | B  |                     |   |   |                                      |
|   | C  |                     |   |   |                                      |



**Fig. 7** Simulated and experimental  $i_{xx}$  before and after HAPF compensation at different operation regions

Simulated results:

- a Before compensation
- b Non-linear region ( $H = 1.25$  A)
- c Quasi-linear region ( $H = 0.50$  A)
- d Linear region ( $H = 0.156$  A)

Experimental results:

- e Before compensation
- f Non-linear region ( $H = 1.25$  A)
- g Quasi-linear region ( $H = 0.50$  A)
- h Linear region ( $H = 0.156$  A)

Fig. 6h. Table 4 illustrates that  $H = 0.156$  A yields the best compensation performance among the three operation regions. The system DPF has been compensated close to unity,  $\text{THD}_{i_{sx}} = 9.8\%$  and  $\text{THD}_{V_x} = 3.0\%$  for the worst phase,  $i_{sn} = 0.64 A_{\text{rms}}$ , in which all the compensated results satisfy the international standards [18, 19]. From Table 4, the system average operating switching frequency is 3.1 kHz.

Comparing Table 4 with Table 3, there are differences between simulation and experimental results, which are actually due to the difference of component parameters, the resolution of the transducers, the digital computation error and the noise in the experiment. Moreover, those factors will affect the HAPF compensation performances significantly in linear region operation because more precise A/D signals, computation and control for the HAPF are required. In order to improve the performances (reduce the slope error) of the HAPF experimental prototype, the hysteresis band  $H$  value can be further reduced.

The experimental results as in Figs. 6e–h and Table 4 agreed with the simulation results as in Figs. 6a–d and Table 3, which verified the hysteresis linearisation study for HAPF under three-phase four-wire power quality compensator application. In addition, the HAPF can compensate the system DPF to unity among the three operation regions. However, only when the HAPF is operating at linear operation region, the compensated  $\text{THD}_{i_{sx}}$  and  $\text{THD}_{V_x}$  can satisfy the international standards [18, 19].

Figs. 7e–f depict the experimental three-phase system current  $i_{sx}$  before and after compensation, in which the results are consistent with the simulation results as shown in Figs. 7a–d. And the current harmonics and neutral current can only be significantly compensated at HAPF linear operation region. When the HAPF operates at linear region, it can obtain acceptable current quality compensation performances (Tables 3 and 4) with an average power loss of about 4.0% of the loading power consumption and a fast response time of less than 10 ms.

## 5 Conclusion

In this study, the characteristic and linear requirement of hysteresis PWM control for a LC-coupling HAPF are investigated and discussed. On the basis of the modeling, the non-linear phenomenon and linearisation process for the HAPF inverter current slope are studied, explored and compared with that of the linear APF. Two limits quasi-linear limit and linear limit are proposed, which separate the HAPF into non-linear, quasi-linear and linear operation regions. In order to ensure the controllability of using the conventional hysteresis control and to avoid obtaining unexpected trigger signals for the switching devices, the hysteresis band and sampling time should satisfy the linear limit requirement. Single-phase simulation and experimental results are presented to verify the hysteresis linearisation study of the HAPF and compared with that of the APF. Finally, representative simulation and experimental results of HAPF under three-phase power quality compensation system are given to prove the validity of the hysteresis linearisation study.

## 6 Acknowledgment

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## 7 References

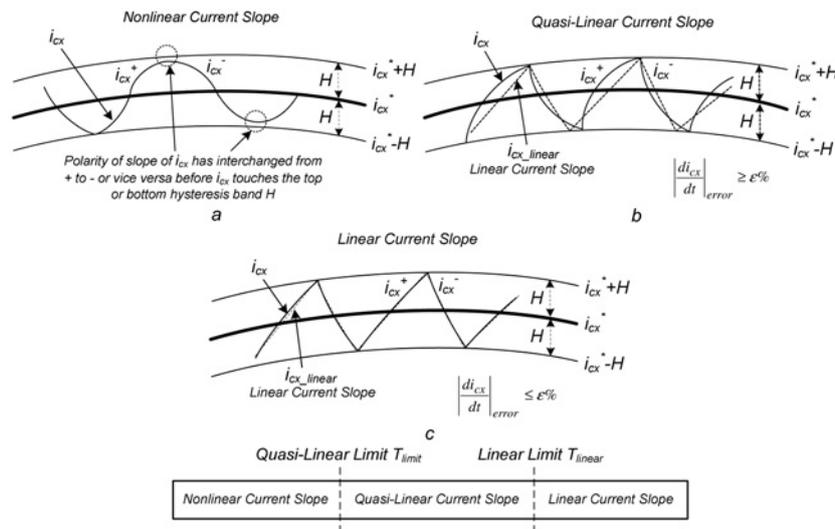
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## 8 Appendix

### 8.1 Non-linearity of HAPF inverter current slope

The definitions and diagrams for the HAPF operating in (i) non-linear region, (ii) quasi-linear region and (iii) linear region under hysteresis control are given below:

1. *Non-linear region*: The slope of the HAPF inverter current  $i_{cx}$  is defined as 'non-linear' if its slope polarity has interchanged from positive to negative or vice versa before  $i_{cx}$  touches the top or bottom hysteresis band  $H$  (within each switching interval), as shown in Fig. 8a.  $i_{cx}^*$  is the reference inverter current.



**Fig. 8** Slope of the HAPF inverter current  $i_{cx}$  under hysteresis PWM control

- a Non-linear region
- b Quasi-linear region
- c Linear region

2. *Quasi-linear region:* The slope of the HAPF inverter current  $i_{cx}$  is defined as ‘quasi-linear’ if its slope polarity changes only when  $i_{cx}$  touches the top or bottom hysteresis band  $H$ . Within each switching on or off interval, the absolute percentage error of its current slope compared with a linear one is larger than  $\epsilon\%$  ( $|(di_{cx})/dt|_{error} > \epsilon\%$ ), as shown in Fig. 8b.

3. *Linear region:* The slope of the HAPF inverter current  $i_{cx}$  can be treated as ‘linear’ if its slope polarity changes only when  $i_{cx}$  touches the top or bottom hysteresis band  $H$ . Within each switching on or off interval, the absolute percentage error of its current slope compared with a linear one is less than or equal to  $\epsilon\%$  ( $|(di_{cx})/dt|_{error} \leq \epsilon\%$ ), as shown in Fig. 8c.

When the coupling part internal resistance is small,  $R_{c1} \simeq 0$ , the HAPF will be an underdamped system. Combining (4) and (6), the complete response of coupling capacitor voltage  $v_{Cc1x(t)}$  can be derived. Then the HAPF inverter current  $i_{cx}$  can be deduced by taking derivative of  $v_{Cc1x(t)}$  via (6). After that, taking derivative of  $i_{cx}$  will yield the rate of change of inverter current ( $di_{cx}/dt$ ). As the initial condition of  $i_{cx}$  will not affect the non-linear nature of the inverter current slope, it can be simply treated as zero. Substitute this initial condition into ( $di_{cx}/dt$ ) during each switching interval, this yields

$$\frac{di_{cx}(t)}{dt} = -\omega_r^2 C_{c1} B_1 A \tag{9}$$

where  $t$  represents each switching on or off interval ( $t = t_{on}$  or  $t_{off}$ ),  $\omega_r = (1/\sqrt{L_{c1}C_{c1}})$ ,  $B_1$  is a real number and kept constant at each switching interval, and  $A = \cos\omega_r t$ . In order to prevent the current slope from being non-linear, the polarity of (9) should be kept unchanged within each switching interval as shown in Fig. 8. However, due to the non-linear term  $A = \cos\omega_r t$ , its polarity may have interchanging within each switching on or off interval. In addition, the non-linear term is mainly affected by the coupling part ( $C_{c1}$ ,  $L_{c1}$ ) and the switching frequency  $f_{sw}$ . As the coupling part is designed basing on the reactive

power consumption and the dominant harmonic current of the loading, the effect of changing the coupling part values will not be studied. As a result, only the effect of changing the switching frequency will be investigated in this paper. In the following, the quasi-linear limit  $T_{limit}$  and linear limit  $T_{linear}$  that divides the HAPF into non-linear, quasi-linear and linear regions will be determined based on a constant reference inverter current  $i_{cx}^*$  ( $di_{cx}^*/(dt) = 0$ ) assumption. Then the quasi-linear limit  $H_{limit}$  and linear limit  $H_{linear}$  for the hysteresis band can be obtained correspondingly.

### 8.2 Quasi-linear limit $T_{limit}$ and linear limit $T_{linear}$ for HAPF inverter current slope

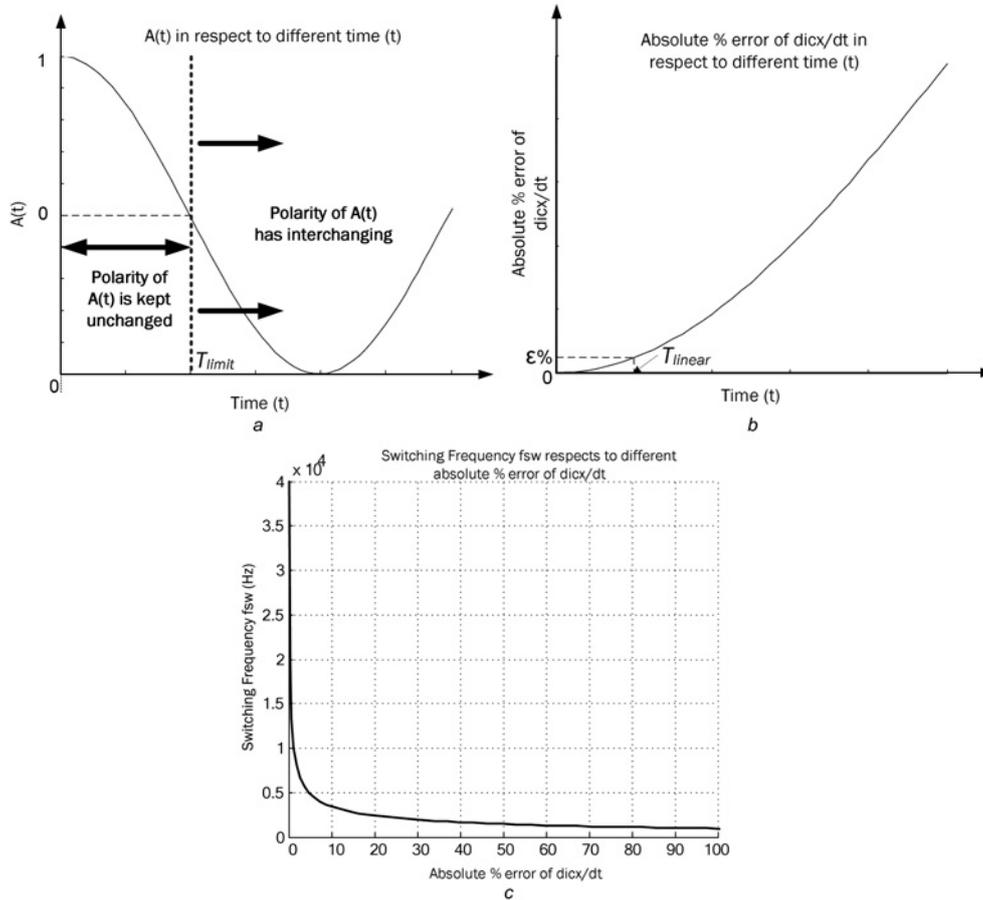
From Fig. 8, the quasi-linear limit  $T_{limit}$  and linear limit  $T_{linear}$  will be deduced based on one switching interval ( $t_{on}$  or  $t_{off}$ ) analysis. As the hysteresis control method yields asymmetric switching on and off intervals during operation, by choosing the switching frequency  $f_{sw} = 1/(t_{on} + t_{off}) \geq 1/2T_{limit}$  or  $f_{sw} \geq 1/2T_{linear}$ , it is possible that either switching on or off interval cannot satisfy the limit  $T_{limit}$  or  $T_{linear}$ . As a result, in order to ensure that each switching interval satisfies the limit, the switching frequency should be chosen as  $f_{sw} \geq 1/T_{limit}$  or  $f_{sw} \geq 1/T_{linear}$ . As a result, once the limits are determined, the inverse of these limits are treated as the quasi-linear limit and linear limit for the switching frequency  $f_{sw}$ .

Fig. 9a illustrates the graphs of non-linear term  $A$  at different time  $t$  and  $R_{c1} \simeq 0$ . From Fig. 9a, in order to prevent the HAPF current slope from being non-linear, the switching frequency  $f_{sw}$  and sampling time  $T$  can be chosen as

$$f_{sw} \geq \frac{1}{T_{limit}}, \quad T \leq T_{limit} \quad \text{and} \quad T \leq \frac{1}{f_{sw}} \tag{10}$$

where  $T_{limit}$  is the quasi-linear limit for each switching on or off interval.

From (9), the absolute error of the HAPF inverter current slope ( $|(di_{cx})/dt|_{error}$ ) compared with a linear one can be



**Fig. 9** Graphs of  $A = \cos\omega t$ ,  $|(di_{cx}/dt)_{error}$  and  $f_{sw}$  against  $|(di_{cx}/dt)_{error}$   
 a  $A = \cos\omega t$  at different time  $t$  and  $R_{c1} \approx 0$   
 b  $|(di_{cx}/dt)_{error}$  at different time  $t$  and  $R_{c1} \approx 0$   
 c  $f_{sw}$  against  $|(di_{cx}/dt)_{error}$

expressed as

$$\left| \frac{di_{cx}}{dt} \right|_{error} = |1 - \cos \omega_p t| \quad (11)$$

From Fig. 8c, once  $(|(di_{cx})/dt|_{error} \leq \epsilon\%)$  is satisfied, the inverter current slope can be approximately treated as linear. From Fig. 9b, if each switching interval is less than  $T_{linear}$ , it can satisfy the HAPF linear region definition  $(|(di_{cx})/dt|_{error} \leq \epsilon\%)$ . In order to obtain a linear inverter current slope, the switching frequency  $f_{sw}$  and sampling time  $T$  can be chosen as

$$f_{sw} \geq \frac{1}{T_{linear}}, \quad T \leq T_{linear} \quad \text{and} \quad T \leq \frac{1}{f_{sw}} \quad (12)$$

where  $T_{linear}$  is the linear limit for each switching on or off interval.

From Fig. 8, if the current slope error limit  $\epsilon$  is set to be large, the HAPF will result in low switching frequency and loss, but with a large operational error. When  $\epsilon$  is set close to zero, it is impractical due to a high switching frequency requirement for the switching devices. Fig. 9c shows the switching frequency  $f_{sw}$  requirement in respect to different absolute percentage error  $(|(di_{cx})/dt|_{error})$  of the current slope. From Fig. 9c, when  $(|(di_{cx})/dt|_{error})$  is desired to be decreased a little bit more from 5%, the switching

frequency requirement will exponentially increase a lot. In this study,  $\epsilon$  is set at 5% because its corresponding switching frequency is within limitation of the switching devices. Moreover, the HAPF compensation performances under  $\epsilon = 5\%$  consideration are acceptable as verified by the simulation and experimental results in Section 4.

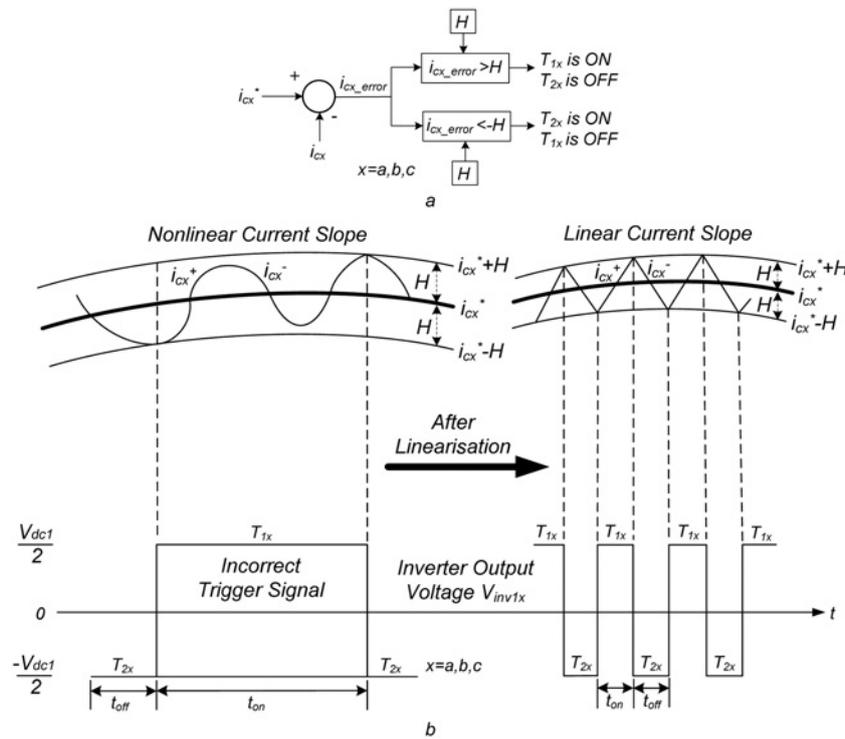
### 8.3 Relationship among hysteresis band $H$ , switching frequency $f_{sw}$ and dc-link voltage $V_{dc1}$ of linearised HAPF

Fig. 10 shows the hysteresis current-controlled PWM for HAPF: Fig. 10a switching scheme block diagram for one phase and Fig. 10b current and voltage waveforms before and after linearisation. From Fig. 10, the relationship among the switching frequency  $f_{sw}$ , hysteresis band  $H$ , dc-link voltage  $V_{dc1}$  under HAPF linear region can be derived. After simplification, the hysteresis band  $H$  value under a fixed switching frequency  $f_{sw}$  can be approximately expressed as

$$H \approx \frac{V_{dc1}}{8L_{c1}f_{sw}} \left[ 1 - \frac{4L_{c1}^2}{V_{dc1}^2} m^2 \right] \quad (13)$$

where  $m = (di_{cx}^*/dt)$  is the slope of the reference inverter current  $i_{cx}^*$ .

As the reference inverter current is assumed to be a constant value during the previous linearisation analysis,



**Fig. 10** Hysteresis current-controlled PWM for HAPF  
 a Switching scheme block diagram for one phase  
 b Current and voltage waveforms before and after linearisation

(13) becomes

$$H = \frac{V_{dc1}}{8L_{c1}f_{sw}} \quad (14)$$

Substituting  $f_{sw} = 1/T_{limit}$  or  $1/T_{linear}$  into (14), this yields the

quasi-linear limit  $H_{limit}$  or linear limit  $H_{linear}$  for the hysteresis band. Under constant reference inverter current  $i_{cx}^*$  assumption, the system operating switching frequency under  $H_{limit}$  or  $H_{linear}$  will be fixed. When  $i_{cx}^*$  is not a constant value, (14) will yield the HAPF maximum operating switching frequency under a fixed hysteresis band.